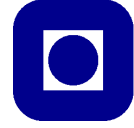


TFY4245/FY8917 Solid State Physics, Advanced Course

NTNU

Problemset 9



Institutt for fysikk

Problem 1

Consider free electrons in 3D moving in the presence of a uniform magnetic field $B = B\hat{z}$. The eigenenergies are

$$E_{n,k_x,k_y,k_z} = \hbar\omega_c(n + 1/2) + \hbar^2 k_z^2 / 2m. \quad (1)$$

(a) Derive the expression for density of states $D(\epsilon)$ (per spin) written down in the lectures

$$D(\epsilon) = \frac{1}{\pi\hbar} \sqrt{\frac{m}{2}} \frac{N_L}{L^2} \sum_{n=0}^{\infty} \frac{\Theta(\epsilon - [n + 1/2]\hbar\omega_c)}{\sqrt{\epsilon - [n + 1/2]\hbar\omega_c}} \quad (2)$$

by using that the density of states is a sum of the density of states for a 1D electron gas shifted to the minimum energies $\hbar\omega_c(n + 1/2)$, according to the above expression for the eigenenergies.

We now want to compute the total zero-temperature energy of the system, E_{tot} , which enables determination of *e.g.* the magnetic susceptibility. Define the following functions

$$\begin{aligned} P_1(x) &\equiv \int_0^x d\epsilon D(\epsilon), \\ P_2(x) &\equiv \int_0^x d\epsilon P_1(x). \end{aligned} \quad (3)$$

(b) Write down an expression for the total energy of the system expressed as an integral over $D(\epsilon)$ and perform this integral to show that

$$E = \mu N - 2VP_2(\mu). \quad (4)$$

Assuming the experimentally relevant regime $\hbar\omega_c \ll \mu$, the sum over n in $P_2(\mu)$ can be replaced by an integral. Use the Poisson summation formula

$$\sum_{n=0}^{\infty} f(n + 1/2) = \int_0^{\infty} dx f(x) + 2 \sum_{s=0}^{\infty} (-1)^s \int_0^{\infty} dx f(x) \cos(2\pi s x) \quad (5)$$

and

$$\sum_{s=1}^{\infty} \frac{(-1)^s}{s^2} = -\frac{\pi^2}{12} \quad (6)$$

to show that

$$P_2 \simeq \frac{1}{\pi\hbar} \sqrt{\frac{m}{2}} \frac{N_L}{L^2} \frac{4}{3} \left[\frac{2}{5} \frac{\mu^{5/2}}{\hbar\omega_c} - \frac{1}{10\sqrt{2}} (\hbar\omega_c)^{3/2} - \frac{1}{16} \hbar\omega_c \sqrt{\mu} \right] \quad (7)$$

when you neglect terms in P_2 which are rapidly oscillating in the limit $\mu/\hbar \rightarrow \infty$, corresponding to the low-field limit $B \rightarrow 0$.

(c) Write out the resulting expression for E to lowest order in B to show that

$$E = E(B = 0) + \kappa B^2 \quad (8)$$

and identify an expression for κ .

Problem 2

When expressing operators in real space in terms of a Fourier-series over momentum-space operators, we frequently encounter summations over momentum and/or lattice coordinate. We have seen this both for fermions in a tight-binding model and for magnons in a Heisenberg model.

Prove that

$$\sum_k e^{ik(r_j - r_{j'})} = N\delta_{jj'} \quad (9)$$

when the components of the wavevectors take values $2\pi m/L$ where m are integers that take values $-N/2, \dots, N/2 - 1$ so that the wavevector lies in the 1st BZ $[-\pi/a, \pi/a)$ and $j = 1, 2, \dots, N$ for a 1D system. The above can be straightforwardly generalized to 2D and 3D.