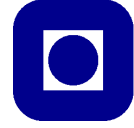


## TFY4245/FY8917 Solid State Physics, Advanced Course

NTNU

## Problemset 5



Institutt for fysikk

**Problem 1**

(a) Derive the exact zero-temperature expression for the static Lindhard function

$$\chi_0(\mathbf{q}) = \frac{1}{V} \sum_{\mathbf{k}, s} \frac{f_{\mathbf{k}} - f_{\mathbf{k}-\mathbf{q}}}{\epsilon_{\mathbf{k}} - \epsilon_{\mathbf{k}-\mathbf{q}}} \quad (1)$$

at  $\mathbf{q} \neq 0$  for a 3D system of free electrons. Here,  $f_{\mathbf{k}}$  is the Fermi-Dirac distribution function and  $\epsilon_{\mathbf{k}} = \hbar^2 \mathbf{k}^2 / 2m$  is the dispersion for free electrons.

(b) What is the corresponding dielectric function and what is its asymptotic value when  $q = |\mathbf{q}| \rightarrow 0$ ? How do you interpret this limit?

(c) When a point-charge  $en_a(\mathbf{r}) = en_{a0}\delta(\mathbf{r})$  is placed in a metal, it induces Friedel oscillations in the induced electron density  $\delta n(\mathbf{r})$ . Use the expression for  $\epsilon(\mathbf{q}, 0)$  above to compute the total electron charge  $\delta Q$  displaced around the point charge from

$$\delta Q = e\delta n = e \int d^3r \delta n(\mathbf{r}) \quad (2)$$

when we know from the Poisson equation that the Fourier-transform of  $\delta n(\mathbf{r})$  is given by

$$e\delta n(\mathbf{q}) = \left[ \frac{1}{\epsilon(\mathbf{q}, 0)} - 1 \right] n_a(\mathbf{q}, 0). \quad (3)$$

Comment on the physical interpretation of the result obtained for  $\delta Q$ .

**Problem 2**

Consider the Frölich Hamiltonian describing a free electron and phonon gas plus an electron-phonon scattering term:

$$H = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} c_{\mathbf{k}}^{\dagger} c_{\mathbf{k}} + \sum_{\mathbf{q}} \hbar \omega_{\mathbf{q}} a_{\mathbf{q}}^{\dagger} a_{\mathbf{q}} + \sum_{\mathbf{k}\mathbf{q}} M_{\mathbf{q}} (a_{-\mathbf{q}}^{\dagger} + a_{\mathbf{q}}) c_{\mathbf{k}+\mathbf{q}}^{\dagger} c_{\mathbf{k}} \quad (4)$$

Here,  $M_{\mathbf{q}} = M_{-\mathbf{q}}^*$  ensures hermiticity. Treat the scattering term as a perturbation and compute the lowest-order correction to the ground-state energy by using perturbation theory. Express this correction in terms of  $M_{\mathbf{q}}, \epsilon_{\mathbf{k}}, \omega_{\mathbf{q}}$  as well as the Fermi-Dirac and Bose-Einstein distribution functions.