TFY4245/FY8917 Solid State Physics, Advanced Course

NTNU

Institutt for fysikk

Problemset 5

Problem 1

(a) Derive the exact zero-temperature expression for the static Lindhard function

$$\chi_0(\boldsymbol{q}) = \frac{1}{V}\sum_{\boldsymbol{k},s} \frac{f_{\boldsymbol{k}} - f_{\boldsymbol{k}-\boldsymbol{q}}}{\varepsilon_{\boldsymbol{k}} - \varepsilon_{\boldsymbol{k}-\boldsymbol{q}}} \tag{1}$$

at $\boldsymbol{q} \neq 0$ for a 3D system of free electrons. Here, $f_{\boldsymbol{k}}$ is the Fermi-Dirac distribution function and $\varepsilon_{\boldsymbol{k}} = \hbar^2\boldsymbol{k}^2/2m$ is the dispersion for free electrons.

(b) What is the corresponding dielectric function and what is its asymptotic value when $q = |\boldsymbol{q}| \to 0$? How do you interpret this limit?

(c) When a point-charge $en_a(r) = en_{a0}\delta(\boldsymbol{r})$ is placed in a metal, it induces Friedel oscillations in the induced electron density $\delta n(\boldsymbol{r})$. Use the expression for $\varepsilon(\boldsymbol{q},0)$ above to compute the total electron charge δQ displaced around the point charge from

$$\delta Q = e\delta n = e\int d^3r\,\delta n(\boldsymbol{r}) \tag{2}$$

when we know from the Poisson equation that the Fourier-transform of $\delta n(\boldsymbol{r})$ is given by

$$e\delta n(\boldsymbol{q}) = \left[\frac{1}{\varepsilon(\boldsymbol{q},0)} - 1\right] n_a(\boldsymbol{q},0). \tag{3}$$

Comment on the physical interpretation of the result obtained for δQ.

Problem 2

Consider the Frölich Hamiltonian describing a free electron and phonon gas plus an electron-phonon scattering term:

$$H = \sum_{\boldsymbol{k}} \varepsilon_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}} c_{\boldsymbol{k}} + \sum_{\boldsymbol{q}} \hbar\omega_{\boldsymbol{q}} a^\dagger_{\boldsymbol{q}} a_{\boldsymbol{q}} + \sum_{\boldsymbol{kq}} M_{\boldsymbol{q}}(a^\dagger_{-\boldsymbol{q}} + a_{\boldsymbol{q}}) c^\dagger_{\boldsymbol{k}+\boldsymbol{q}} c_{\boldsymbol{k}} \tag{4}$$

Here, $M_{\boldsymbol{q}} = M^*_{-\boldsymbol{q}}$ ensures hermiticity. Treat the scattering term as a perturbation and compute the lowest-order correction to the ground-state energy by using perturbation theory. Express this correction in terms of $M_{\boldsymbol{q}}, \varepsilon_{\boldsymbol{k}}, \omega_{\boldsymbol{q}}$ as well as the Fermi-Dirac and Bose-Einstein distribution functions.