TFY4245/FY8917 Solid State Physics, Advanced Course

Problemset 4

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Problem 1

(a) In the lectures, we considered the basic thermodynamic properties of metals. Consider a free electron gas where the electron occupation is described by the Fermi-Dirac distribution in equilibrium. Derive an expression for the internal energy of the gas

$$U(T) = \int_0^\infty dE\, E\, N(E) f(E) \tag{1}$$

up to and including second order in temperature.

(b) What is the difference between the internal energy U and the free energy F of an electron gas in equilibrium?

Problem 2

We proved the stability of the metallic state

$$|\Psi_0\rangle = \prod_{|\boldsymbol{k}|\leq k_F} \prod_s c^\dagger_{\boldsymbol{k},s} |0\rangle \tag{2}$$

in the lecture notes.

(a) Explain in words how the electrons are distributed in various quantum states when the system is described by $|\Psi_0\rangle$.

(b) When evaluating the energy resulting from the Coulomb repulsion between the electrons in the above proposed metallic ground state, we made use of the following expectation value:

$$\sum_{ss'} \langle \Psi_0 | \Psi^\dagger_s(\boldsymbol{r}) \Psi^\dagger_{s'}(\boldsymbol{r}') \Psi_{s'}(\boldsymbol{r}') \Psi_s(\boldsymbol{r}) | \Psi_0 \rangle = n^2 - G(\boldsymbol{r} - \boldsymbol{r}') \tag{3}$$

where n is the electron density and

$$G(\boldsymbol{r}) = \frac{9n^2}{2} \left(\frac{k_F|\boldsymbol{r}| \cos(k_F|\boldsymbol{r}|) - \sin(k_F|\boldsymbol{r}|)}{(k_F|\boldsymbol{r}|)^3} \right). \tag{4}$$

Derive this equation.