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Examination paper for TFY4245 / FY8917 Solid State Physics, Advanced Course

Date: May 13th, 2025
Time: 15:00-19:00

Course contact: Jacob Linder
Present at the exam location: No

Permitted examination support material: no hand-written support. The mathematical formula collection by Karl Rottmann is permitted (Norwegian or German version are both fine). If the student does not know Norwegian or German, an equivalent collection of mathematical formula in a different language may be used.

OTHER INFORMATION

Read the questions carefully and make your own assumptions. In your answers, explain clearly what assumptions you have made and how you have understood or limited the assignment

If there are direct errors or omissions in the assignment set and you cannot make your own assumptions, please refer to the information about complaints regarding formal errors on the NTNU website “Explanation of grades and appeals”.

SPECIFIC INFORMATION FOR YOUR COURSE

Hand drawings: For question 1,2,3 you are meant to answer on handwritten sheets. Other questions must be answered directly in Inspera. At the bottom of the question, you will find a seven-digit code. Fill in this code in the top left corner of the sheets you wish to submit.

We recommend that you do this during the exam. If you require access to the codes after the examination time ends, click “Show submission”.

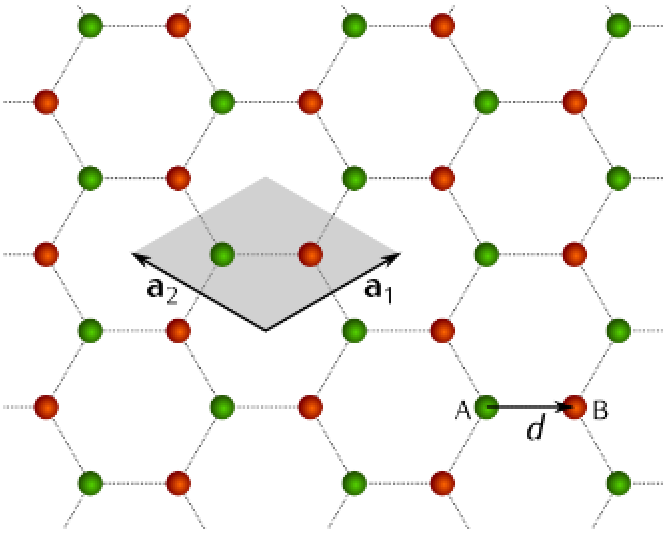
You are responsible for filling in the correct codes on the handwritten sheets. Therefore, read the cover sheet carefully. The Examination Office cannot guarantee that incorrectly completed sheets will be added to your assignment.

Weighting: The maximum number of points that can be achieved is indicated in the title of each problem.

Withdrawing from the exam: If you wish to submit a blank test/withdraw from the exam for another reason, go to the menu in the top right-hand corner and click “Submit blank”. This cannot be undone, even if the test is still open.

Access to your answers: After the exam, you can find your answers under previous tests in Inspera. Be aware that it may take a working day until any hand-written material is available in “previous tests”.

1 Problem 1 (3 points)



The lattice of graphene, a mono-atomic 2D layer of carbon atoms, is shown in the figure. All carbon atoms are identical and the nearest-neighbor distance is d. Also shown is one possible choice of unit cell. The vectors are $\mathbf{a}_1 = \frac{d}{2}(\sqrt{3}, 1)$ and $\mathbf{a}_2 = \frac{d}{2}(-\sqrt{3}, 1)$.

(a) Draw the Wigner-Seitz cell of graphene and explain precisely how you obtained it.

(b) What are the unit vectors for graphene in reciprocal space?

(c) ARPES is an experimental technique used to measure the energy bands of electrons in a material. Explain briefly the basic physics behind this technique and how it allows one to extract information about electron band energies.

Maximum marks: 3

2 Problem 2 (6 points)

Consider a semiconducting material with a conduction band $\varepsilon_{C,\mathbf{k}}$ and a valence band $\varepsilon_{V,\mathbf{k}}$. Assume that the entire valence band is filled and the conduction band is empty at $T = 0$.

(a)

- Write down the Fock-space ground-state of this system using the second quantized formalism.
- What is the energy of the ground state?

(b)

- Explain what an exciton is and draw a figure where you indicate where the exciton levels are typically located relative the conduction band energy in a semiconductor.
- How do excitons affect the absorption of electromagnetic radiation in a direct-gap semiconductor? Draw a diagram where you indicate how absorption takes place as a function of the frequency of the electromagnetic field in a semiconductor with excitons.

(c) Assume that a system is exposed to an electric field. The linear-response of the expectation value of the quantum mechanical current density operator is then given by

$$\langle \hat{J}(t) \rangle = \int_{-\infty}^{\infty} \sigma(t - t') E(t') dt'$$

- Which consequence does the requirement of causality have for the response function $\sigma(t)$?
- Derive what the relation between current and electric field looks like in frequency-space (see Formula Sheet for definition of Fourier-transformation conventions) under the assumption of time-translational invariance.

Maximum marks: 6

3 Problem 3 (6 points)

Consider a ferromagnetic Heisenberg model in two spatial dimensions described by a Hamiltonian:

$$H = \frac{J}{2}\sum_{i\neq j}\mathbf{S}_i \cdot \mathbf{S}_j \quad = J\sum_{i,\delta}[\frac{1}{2}(S_i^+ S_{i+\delta}^- + S_i^- S_{i+\delta}^+) + S_i^z S_{i+\delta}^z]$$

Consider a square lattice and let δ run over half the nearest neighbor vectors so that $\delta \in \hat{\mathbf{x}}, \hat{\mathbf{y}}$. Let $\hbar = a = 1$ where a is the lattice constant. The linearized Holstein-Primakoff (HP) transformation reads:

$$S_j^+ = \sqrt{2S}a_j S_j^- = \sqrt{2S}a_j^\dagger S_j^z = S - a_j^\dagger a_j$$

(a) Insert the linearized HP transformation and write down the resulting expression for H.

(b) Assume translational invariance in a model with periodic boundary conditions and use an inverse Fourier-transformation

$$a_i = \sqrt{\frac{1}{N}}\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}_i} a_{\mathbf{k}}$$

where N is the number of lattice sites to write down the resulting Hamilton-operator in momentum-space. You may find it useful to know that $\sum_i e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_i} = N\delta_{\mathbf{k},\mathbf{k}'}$.

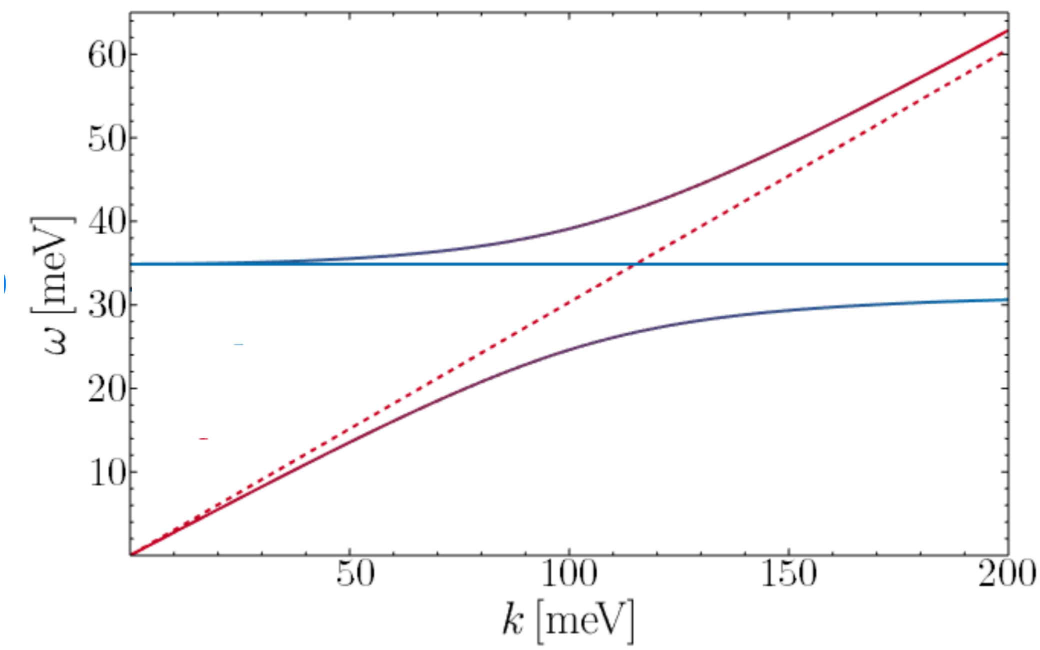
Which values can k_x, k_y take and how do you derive this criterion? You may for simplicity assume that the number of lattice sites in both the x and y direction is an even number.

(c) The Hamilton-operator you derive in (b) should consist of a constant term that is independent on the magnon operators and a term containing a summation over all momenta and which depends on the magnon operators. What is the physical significance of these two terms, individually?

(d) Let $M = \frac{1}{N}\sum_i \langle S_i^z \rangle$ be the order parameter for the system. Use this definition to write down an analytical expression for M expressed in terms of S, temperature T, and the magnon dispersion relation $\omega_{\mathbf{k}}$.

Maximum marks: 6

4 Problem 4 (6 points)



(a) Explain in an accurate manner what it means that U(1) symmetry is spontaneously broken in a superconductor.

(b) When should the Landau free energy contain a linear term in the order parameter? Give an example of a physical system where this happens.

(c) What is the difference between a polar and noncentrosymmetric unit cell in a crystal?

(d) Describe how phonons can cause a structural phase transition between two crystal structures.

(e) The figure shows the dispersion relations for an optical phonon, a polariton, and a photon. Which line corresponds to which type of excitation? Which property of the polaritons allows them to be measured experimentally using reflectance in optical measurements?

Fill in your answer here

Format B I U x_2 x^2 I_x

Words: 0

Maximum marks: 6

Formula sheet

Some of the following formula will be useful during the exam. The student is expected to know the meaning of the symbols and the context for the equations.

Lattice vectors:

$$\mathbf{a}_i \cdot \mathbf{b}_j = 2\pi\delta_{ij} \text{ with } i, j \in \{1, 2, 3\}, \mathbf{b}_i = 2\pi \frac{\mathbf{a}_j \times \mathbf{a}_k}{\mathbf{a}_i \cdot (\mathbf{a}_j \times \mathbf{a}_k)}$$

Fourier-transformation:

$$f(\omega) = \int dt e^{i\omega t} f(t) \text{ and } f(t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} f(\omega).$$

Boson-commutators:

$$[a_\lambda, a^\dagger_{\lambda'}] = \delta_{\lambda,\lambda'} \qquad [a^\dagger_\lambda, a^\dagger_{\lambda'}] = [a_\lambda, a_{\lambda'}] = 0.$$

Fermi-Dirac and Bose-Einstein distribution:

$$f_{\text{F-D}}(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}+1}, \; f_{\text{B-E}}(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}-1}$$