TFY4205 Quantum Mechanics II
Problemset 8 fall 2022

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Problem 1 (Landau levels)

We have previously derived the eigenvalues

$$E = (n+\frac{1}{2})\hbar\omega_c + \frac{\hbar^2 k_z^2}{2m},\ n = 0,1,2,\dots \tag{1}$$

for the Hamilton operator

$$\hat{H} = \frac{(\hat{\boldsymbol{p}} - q\boldsymbol{A})^2}{2m} \tag{2}$$

by using the Landau-gauge $\boldsymbol{A} = (-By, 0, 0)$ which describes a constant magnetic field B applied along the z-axis. The electron charge is $q = -e$ and we defined $\omega_c = eB/m$. This result was derived using eigenfunctions on the form $\psi = \mathrm{e}^{\mathrm{i}k_x x + \mathrm{i}k_z z}\phi(y)$, motivated by the fact that the Hamilton-operator in the chosen gauge commutes with $\hat{p}_x$ and $\hat{p}_z$ (so that both momentum in the x and z direction are good quantum numbers).

Consider now a gauge choice $\boldsymbol{A} = \frac{B}{2}(-y, x, 0)$. This produces the same magnetic field as above. Derive the eigenvalues in this gauge choice. How are they different from the eigenvalues above?

Hint to solve this problem: after writing out the Hamilton operator in this so-called symmetric gauge, see if you can identify quantities which act just like creation and annihilation operators and write the Hamilton operator in terms of these. In particular, since the new gauge choice includes both x and y, it could be useful to work with a complex variable $z = x - \mathrm{i}y$. Defining the annihilation operator $a = \sqrt{2}(\partial_{\bar{z}} + z/4)$ where $\bar{z}$ is the complex conjugate of z could then get you started.