TFY4205 Quantum Mechanics II
Problemset 7 fall 2022

NTNU

Institutt for fysikk

Problem 1 (scattering with identical particles)
A fermion is scattered by an identical fermion at low energy. The interaction is assumed to be spherically symmetric with finite range. In the experiment, both fermions have their spin oriented along the same direction. What is the dominant angle dependence of the differential scattering cross section in the center of mass coordinate system?

Problem 2 (gauge invariance)
The electromagnetic potentials

$$\Phi = 0, \boldsymbol{A} = -\frac{et}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{r^3} \tag{1}$$

give the following physically observable fields

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} = 0,\ \boldsymbol{E} = -\nabla\Phi - \partial_t \boldsymbol{A} = \frac{e}{4\pi\varepsilon_0}\frac{\boldsymbol{r}}{r^3}. \tag{2}$$

These are the electrostatic fields from a positive charge $+e$ located at the origin $\boldsymbol{r} = 0$. The Schrodinger equation for a negative charge $-e$ existing in the fields of a positive charge may therefore be written as

$$\mathrm{i}\hbar\partial_t \Psi(\boldsymbol{r},t) = \frac{(\boldsymbol{p}+e\boldsymbol{A})^2}{2m}\Psi(\boldsymbol{r},t), \tag{3}$$

where $\boldsymbol{p}$ is the momentum operator. This is perhaps a different Schrodinger equation for a charge in a Coulomb potential from the one we are used to!

Therefore, use a gauge-transformation to show that the above equation gives exactly the same physics as the standard Schrodinger equation for this scenario.