TFY4205 Quantum Mechanics II
Problemset 5 fall 2022

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Problem 1 (adiabatic theorem, Berry phase)
The adiabatic theorem in quantum mechanics states the following.

Consider a Hamilton operator $\hat{H}(t)$ which has a discrete and non-degenerate spectrum. The instantaneous eigenstates $|\psi_n(t)\rangle$ of $\hat{H}(t)$ satsify

$$\hat{H}(t)|\psi_n(t)\rangle = E_n(t)|\psi_n(t)\rangle \tag{1}$$

at any time t and form an orthonormal set. If at $t = 0$ the system is in eigenstate $|\psi_i(0)\rangle$, so that $|\Psi(0)\rangle = |\psi_i(0)\rangle$, then the theorem dictates that at a later time t the system state is given by

$$|\Psi(t)\rangle = \mathrm{e}^{\mathrm{i}\theta(t)+\mathrm{i}\gamma(t)}|\psi_i(t)\rangle \tag{2}$$

so long as $\hat{H}(t)$ changes sufficiently slowly. In the lectures, we derived that the dynamical phase is

$$\theta(t) = -\frac{1}{\hbar}\int_0^t dt' E_i(t') \tag{3}$$

while the Berry phase is

$$\gamma(t) = \mathrm{i}\int_0^t \langle\psi_i(t')|\dot{\psi}_i(t')\rangle dt'. \tag{4}$$

Using the above results, consider now the following problem. Imagine we have a different Hamilton operator $\hat{\hat{H}}(t)$ which is related to the original Hamiltonian as follows

$$\hat{\hat{H}}(t) = \hat{H}[g(t)] \tag{5}$$

in a time interval $0 < t < t_1$. The function $g(t)$ satisfies $g(0) = 0$ and $g(t_1) = t_1$. What this means in practice is that the two Hamiltonians both have the same values at $t = 0$ and $t = t_1$, but they are not equal at intermediate times. In effect, they can have a different rate of change between $t = 0$ and $t = t_1$.

Which consequence does this have for the dynamical phase and Berry phase of the system evaluated at $t = t_1$? In other words, is the dynamical phase evaluated at $t = t_1$ obtained using $\hat{\hat{H}}(t)$ different from the dynamical phase evaluated at $t = t_1$ obtained using $\hat{H}(t)$, and what about the Berry phase?

Problem 2 (sudden approximation)
Consider a 1D infinite square well with walls at $x = 0$ and $x = L$. A particle resides in the groundstate of this system for $t < 0$. At $t = 0$, we suddenly increase the width of the well to $2L$. Find the probability that the particle will be found in the nth stationary state of the expanded well at $t > 0$.