TFY4205 Quantum Mechanics II
Problemset 11 fall 2022

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Problem 1 (entanglement of W state)
Show that the state

$$|W_n\rangle = \frac{1}{\sqrt{n}}\big(|0...001\rangle + |0...010\rangle + |0...100\rangle + ... + |1...000\rangle\big) \tag{1}$$

is an entangled state whenever $n > 1$.

Problem 2 (observables in an entangled state and CHSH inequality)
Alice and Bob share a two-qubit state which is an imperfect entangled state:

$$\rho = p\frac{I}{4} + (1-p)|\psi^{AB}\rangle\langle\psi^{AB}| \tag{2}$$

where we have defined

$$|\psi^{AB}\rangle = \frac{1}{\sqrt{2}}\big(|0^A\rangle|1^B\rangle - |1^A\rangle|0^B\rangle\big). \tag{3}$$

If $p = 0$, we would have had a maximally entangled singlet state. The observables that they can measure on their qubit is either the σ_z value or a combination of σ_z and σ_x. Specifically, the obervables being measured by Alice and Bob are:

$$P : \sigma_z^A \tag{4}$$
$$Q : \cos\big(\frac{\pi}{4}\big)\sigma_z^A + \sin\big(\frac{\pi}{4}\big)\sigma_x^A \tag{5}$$
$$R : \sigma_z^B \tag{6}$$
$$S : \cos\big(\frac{\pi}{4}\big)\sigma_z^B - \sin\big(\frac{\pi}{4}\big)\sigma_x^B \tag{7}$$

where $\sigma_z^A \equiv \sigma_z^A \otimes I^B$ is an observable on Alice's qubits only, and so on.

Calculate the CHSH-like correlation function $E(P,R) + E(Q,R) + E(P,S) - E(Q,S)$ for this state where $E(X,Y)$ is the expectation value of $X \otimes Y$. For which values of p is the CHSH inequality violated?