

NTNU Trondheim, Institutt for fysikk

Examination for FY3464 Quantum Field Theory I

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Allowed tools: mathematical tables

1. The $\lambda\phi^3$ theory.

Consider the theory of a massive real scalar field ϕ and a $\lambda\phi^3$ self-interaction in $d = 6$ dimensions.

- Write down the Lagrange density \mathcal{L} and explain your choice of signs and pre-factors. (6 pts)
- Write down the corresponding generating functional for disconnected and connected Green functions. How does one obtain connected Green functions? (3 pts)
- Determine the dimension of the field ϕ and of the coupling λ . (3 pts)
- Draw the Feynman diagram(s) and write down the analytical expression for the self-energy $i\Sigma$ (i.e. the loop correction for the free propagator) at order $\mathcal{O}(\lambda^2)$ in momentum space. (4 pts)
- Determine the symmetry factor of $i\Sigma$. (3 pts)
- Calculate the self-energy $i\Sigma$ using dimensional regularisation, split the result into a divergent pole term and a finite remainder. (14 pts)

2. Fermions.

- Define left- and right-chiral fields ψ_L and ψ_R as eigenfunctions of γ^5 . Express

$$\mathcal{L} = \bar{\psi}i\partial\psi - m\bar{\psi}\psi$$

in terms of ψ_L and ψ_R . (7 pts)

- Give an operator which commutes with the (free Dirac) Hamiltonian and can be used to classify the spin states of a fermion. Explain its meaning. (You don't have to calculate the commutator.) (3 pts)

3. Scattering.

Derive the optical theorem

$$2\Im T_{ii} = \sum_n T_{in}^* T_{ni}.$$

Give a physical interpretation of this relation (less than 100 words). (7 pts)

4. Gauge invariance.

Consider a local gauge transformation

$$U(x) = \exp\left[ig \sum_{a=1}^m \vartheta^a(x) T^a\right]$$

which changes a vector of fermion fields ψ with components $\{\psi_1, \dots, \psi_k\}$ as

$$\psi(x) \rightarrow \psi'(x) = U(x)\psi(x).$$

a.) Assume that U are elements of the non-abelian gauge group $SU(n)$ and that $\{\psi_1, \dots, \psi_5\}$ transform with the fundamental representation. What are then the values of n and m ? What is the physical interpretation of m ? (5 pts)

b.) Derive the transformation law of $A_\mu = A_\mu^a T^a$ under a gauge transformation. One way to do this is to require that i) the covariant derivatives transform in the same way as ψ ,

$$D_\mu \psi(x) \rightarrow [D_\mu \psi(x)]' = U(x)[D_\mu \psi(x)].$$

and ii) that the gauge field should compensate the difference between the normal and the covariant derivative, (8 pts)

$$D_\mu \psi(x) = [\partial_\mu + igA_\mu(x)]\psi(x).$$

c.) The non-abelian field-strength $F_{\mu\nu} = F_{\mu\nu}^a T^a$ transforms under a local gauge transformation $U(x)$ as (2 pts)

- $F_{\mu\nu} \rightarrow F'_{\mu\nu} = F_{\mu\nu}$
- $F_{\mu\nu} \rightarrow F'_{\mu\nu} = U(x)F_{\mu\nu}U^\dagger(x)$
- $F_{\mu\nu} \rightarrow F'_{\mu\nu} = U(x)F_{\mu\nu}U^\dagger(x) + \frac{i}{g}(\partial_\mu U(x))\partial_\nu U^\dagger(x)$
- $F_{\mu\nu} \rightarrow F'_{\mu\nu} = F_{\mu\nu} + [D_\mu, A_\nu]$

Some formulas

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}. \quad (1)$$

$$\gamma^5 \equiv \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \quad (2)$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \quad (3)$$

$$\bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0 \quad (4)$$

$$\frac{1}{ab} = \int_0^1 \frac{dz}{[az + b(1-z)]^2}. \quad (5)$$

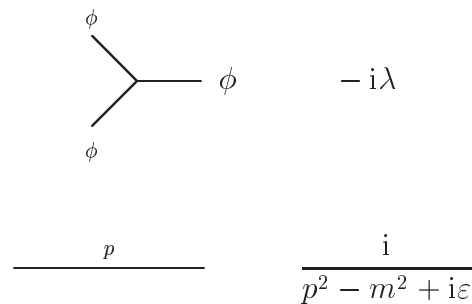
$$\begin{aligned} I(\omega, \alpha) &= \int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} \frac{1}{[k^2 + 2pk + M^2 + i\varepsilon]^\alpha} \\ &= i \frac{(-\pi)^\omega}{(2\pi)^{2\omega}} \frac{\Gamma(\alpha - \omega)}{\Gamma(\alpha)} \frac{1}{[M^2 - p^2 + i\varepsilon]^{\alpha - \omega}}. \end{aligned} \quad (6)$$

$$f^{-\varepsilon/2} = 1 - \frac{\varepsilon}{2} \ln f + \mathcal{O}(\varepsilon^2). \quad (7)$$

$$\Gamma(n+1) = n! \quad (8)$$

$$\Gamma(-n + \varepsilon) = \frac{(-1)^n}{n!} \left[\frac{1}{\varepsilon} + \psi(n+1) + \mathcal{O}(\varepsilon) \right], \quad (9)$$

$$\psi(n+1) = 1 + \frac{1}{2} + \dots + \frac{1}{n} - \gamma, \quad (10)$$



The diagram shows a vertex where two incoming lines labeled ϕ meet and one outgoing line labeled ϕ is produced. To the right of the vertex is the value $-i\lambda$. Below this, a horizontal line labeled p is shown above a propagator expression $\frac{i}{p^2 - m^2 + i\varepsilon}$.