

NTNU Trondheim, Institutt for fysikk

Examination for FY3464 Quantum Field Theory I

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Allowed tools: mathematical tables

1. Spin zero.

Consider a real, scalar field ϕ with mass m and a quartic self-interaction proportional to λ in $d = 4$ space-time dimensions. (4 pts)

a.) Write down the Lagrange density \mathcal{L} , explain your choice of signs and pre-factors (when physically relevant). (4 pts)

b.) Determine the mass dimension of all quantities in the Lagrange density \mathcal{L} . (6 pts)

c.) Draw the Feynman diagrams for $\phi\phi \rightarrow \phi\phi$ scattering at $\mathcal{O}(\lambda^2)$, determine the symmetry factor of these diagrams, and write down the expression for the Feynman amplitude $i\mathcal{A}$ of this process in momentum space. (8 pts)

d.) The one loop correction to the scalar propagator is

$$G^{(2)}(p) = \frac{i}{p^2 - m^2 - \frac{i\lambda}{2}\Delta_F(0) + i\epsilon}. \quad (1)$$

Calculate the self-energy or mass correction $\delta m^2 = \frac{i\lambda}{2}\Delta_F(0)$ in dimensional regularisation (DR). You should end up with something of the form (10 pts)

$$\delta m^2 = \lambda m^2 [a/\epsilon + b + c \ln(\mu^2/m^2)]. \quad (2)$$

e.) What is your interpretation of the dependence of δm^2 on the parameter μ in Eq. (2)? [max. 50 words or one formula without explicit calculation is enough] (4 pts)

2. Spin one-half. Consider a theory of two Weyl fields, a left-chiral field ϕ_L and a right-chiral field ϕ_R , with kinetic energy

$$\mathcal{L}_0 = i\phi_R^\dagger \sigma^\mu \partial_\mu \phi_R + i\phi_L^\dagger \bar{\sigma}^\mu \partial_\mu \phi_L \quad (3)$$

a.) Add a Dirac mass term \mathcal{L}_D . (3 pts)

b.) Find the transformation property of \mathcal{L}_0 and \mathcal{L}_D under parity, $P\mathbf{x} = -\mathbf{x}$. (4 pts)

c.) Add a coupling \mathcal{L}_{int} to the photon A^μ such that the coupling constant is dimensionless. (4 pts)

3. Spin one.

Consider a massless spin-one particle, e.g. the photon A^μ with Lagrange density

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{cl}} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu A_\mu)^2. \quad (4)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

- a.) List the symmetries of \mathcal{L}_{cl} , and of \mathcal{L}_{eff} . (5 pts)
- b.) Derive the corresponding propagator $D_{\mu\nu}(k)$. [You don't have to care how the pole is handled.] (10 pts)
- c.) Write down the generating functionals for disconnected and connected Green functions of this theory. (4 pts)
- d.) How does one obtain connected Green functions from the generating functional? (3 pts)
- e.) What are the two main changes in \mathcal{L}_{cl} and in $\mathcal{L}_{\text{eff}} - \mathcal{L}_{\text{cl}}$ in case of a non-abelian theory? [max. 50 words] (4 pts)

Some formulas

The Pauli matrices are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (5)$$

They satisfy $\sigma^i \sigma^j = \delta^{ij} + i\varepsilon^{ijk} \sigma^k$. Combining the Pauli matrices with the unit matrix, we can construct the two 4-vectors $\sigma^\mu \equiv (1, \boldsymbol{\sigma})$ and $\bar{\sigma}^\mu \equiv (1, -\boldsymbol{\sigma})$.

The Gamma matrices satisfy the Clifford algebra

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu} \quad (6)$$

and are in the Weyl or chiral representation given by

$$\gamma^0 = 1 \otimes \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (7)$$

$$\gamma^i = \sigma^i \otimes i\tau_3 = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad (8)$$

$$\gamma^5 = 1 \otimes \tau_3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (9)$$

$$\psi_L = \frac{1}{2}(1 - \gamma^5)\psi \equiv P_L\psi \quad \text{and} \quad \psi_R = \frac{1}{2}(1 + \gamma^5)\psi \equiv P_R\psi. \quad (10)$$

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu] \quad (11)$$

$$\bar{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0 \quad (12)$$

$$\frac{1}{ab} = \int_0^1 \frac{dz}{[az + b(1-z)]^2}. \quad (13)$$

$$\frac{1}{k^2 + m^2} = \int_0^\infty ds e^{-s(k^2 + m^2)} \quad (14)$$

$$\int_{-\infty}^\infty dx \exp(-x^2/2) = \sqrt{2\pi} \quad (15)$$

$$f^{-\varepsilon/2} = 1 - \frac{\varepsilon}{2} \ln f + \mathcal{O}(\varepsilon^2). \quad (16)$$

$$\Gamma(z) = \int_0^\infty dt e^{-t} t^{z-1} \quad (17)$$

$$\Gamma(n+1) = n! \quad (18)$$

$$\Gamma(-n + \varepsilon) = \frac{(-1)^n}{n!} \left[\frac{1}{\varepsilon} + \psi_1(n+1) + \mathcal{O}(\varepsilon) \right], \quad (19)$$

$$\psi_1(n+1) = 1 + \frac{1}{2} + \dots + \frac{1}{n} - \gamma, \quad (20)$$