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Department of Physics

Examination paper for FY3464/FY8914 Quantum Field Theory

Examination date: 04.06.21

Examination time (from-to): 09:00-13:00

Permitted examination support material: A / All support material is allowed

Academic contact during examination: Jacob Linder
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Technical support during examination: Orakel support services
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If you experience technical problems during the exam, contact Orakel support services as soon as possible before the examination time expires. If you don't get through immediately, hold the line until your call is answered.

OTHER INFORMATION

Make your own assumptions: If a question is unclear/vague, make your own assumptions and specify them in your answer. Only contact academic contact in case of errors or insufficiencies in the question set.

Cheating/Plagiarism: The exam is an individual, independent work. Examination aids are permitted, but make sure you follow any instructions regarding citations. During the exam it is not permitted to communicate with others about the exam questions, or distribute drafts for solutions. Such communication is regarded as cheating. All submitted answers will be subject to plagiarism control. *Read more about cheating and plagiarism here.*

Citations: You should not copy texts or solutions from other sources. Use your own words and understanding of a problem when you write your answer.

Notifications: If there is a need to send a message to the candidates during the exam (e.g. if there is an error in the question set), this will be done by sending a notification in Inspera. A dialogue box will appear. You can re-read the notification by clicking the bell icon in the top right-hand corner of the screen. All candidates will also receive an SMS to ensure that nobody misses out on important information. Please keep your phone available during the exam.

Weighting: The weighting for each problem is indicated in the problem title.

ABOUT SUBMISSION

THIS EXAM CONTAINS QUESTIONS WHERE THE ANSWER SHOULD BE UPLOADED IN PDF OR PICTURE FORMAT.

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30 minutes are added to the examination time to manage the sketches/calculations/files. The additional time is included in the remaining examination time shown in the top left-hand corner.

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Withdrawing from the exam: If you become ill, or wish to submit a blank test/withdraw from the exam for another reason, go to the menu in the top right-hand corner and click “Submit blank”. This cannot be undone, even if the test is still open.

Accessing your answer post-submission: You will find your answer in Archive when the examination time has expired.

1 Problem 1 (5 points)

NB! In all your answers to the questions below, use mostly words, but also a few key equations when needed. You may also insert simple figures/drawings if you like. Your answers do not have to be lengthy in order to get a full score, but be precise in what you write and make sure to discuss the most important points for each question. Use your own words and understanding of a problem when you write your answer.

(a) When evaluating the Feynman diagrams in an interacting quantum field theory, we encountered divergences. These were handled using the concepts of regularization, counterterms, and renormalization. Explain how these concepts are used to remove divergences in Feynman diagrams and how they give rise to renormalized parameters.

(b) What is a running coupling and how does it arise when doing perturbation theory on an interacting quantum field theory?

(c) What are Green functions and why are they useful in quantum field theory? Mention an example of how a Green functions in scalar theory can be physically interpreted and which experimentally observable quantities that can be computed using such Green functions.

Fill in your answer here

Format B I U x_2 x^2 I_x Ω Σ

Words: 0

Maximum marks: 5

2 Problem 2 (4 points)

(a) Consider a Lagrangian density $\mathcal{L}$ describing a complex scalar field:

$$\mathcal{L} = \partial_\mu \phi(x)^* \partial^\mu \phi(x) - m^2 |\phi(x)|^2.$$

Derive an equation of motion for $\phi \text{ and } \phi^*$ by treating them as independent variables. Which of the fields satisfy a Klein-Gordon equation?

(b) Derive the Hamiltonian density $\mathcal{H}$ for the above model expressed in terms of the fields and their canonical momenta.

(c) Show that the normalization condition for a set of single-particle momentum states

$$\langle \boldsymbol{p}_1 | \boldsymbol{p}_2 \rangle = 2E_1 (2\pi)^3 \delta^3(\boldsymbol{p}_1 - \boldsymbol{p}_2).$$

is Lorentz-invariant. Here, E_1 is the relativistic energy for a state with 3-momentum $\boldsymbol{p}_1$.

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Maximum marks: 4

3 Problem 3 (9 points)

Figure part A

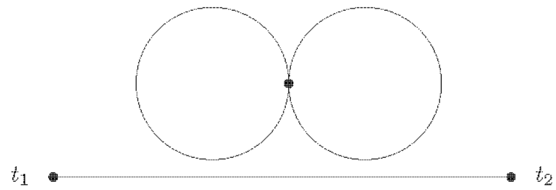
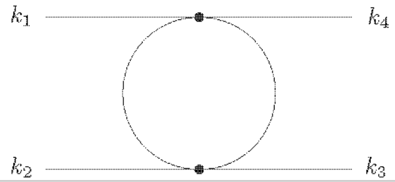


Figure part B



NB! In the questions below where you are asked to explain a concept or procedure, use both words, equations, and figures in your answer. Use your own words and understanding of a problem when you write your answer.

(a) Explain the steps underlying how one uses the partition function Z for a theory such as scalar ϕ^4-theory to obtain the Feynman rules that allows for a perturbative computation of a correlator.

(b) When computing the 2-point correlator in interacting scalar ϕ^4-theory, it is useful to utilize the concept of a self-energy Σ.

First, explain what the self-energy expresses physically.

Then, explain how the full 2-point correlator in the 0+1 dimensional interacting scalar ϕ^4-theory is related to both the 2-point correlator in the non-interacting scalar field case and the self-energy.

(c) Draw all diagrams that contribute to the normalized, interacting 2-point correlator in a 0+1 dimensional ϕ^4-theory up to and including $\mathcal{O}(\lambda)$.

(d) Consider the diagram shown in figure part A which is of $\mathcal{O}(\lambda)$.

Explain why such a diagram does not contribute to the normalized, interacting 2-point correlator in a 0+1 dimensional ϕ^4-theory. Evaluate the complex number associated with this diagram using the Feynman rules of the theory.

(e) When computing 4-point correlators in 3+1 dimensional interacting scalar ϕ^4-theory, one finds a truncated vertex correction diagram of the type shown in figure part B.

First, use the Feynman rules to write down an expression for the complex number associated with this diagram. You do not have to solve any of the integrals in this expression. Explain in particular how you computed the symmetry factor for this diagram.

Then, perform a Feynman parametrization and subsequent Wick rotation in the expression to write it in a new form which includes integration of Euclidean 4D-momentum rather than a Minkowski-space 4-vector. You still do not have to solve any of the integrals in the resulting expression.

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Maximum marks: 9

4 Problem 4 (3 points)

(a) Explain what the *S*-matrix element expresses physically and how it is related to physically observable quantities. Use both words and equations in your answer.

(b) Consider now the Dirac field ψ and the Dirac equation for a free Dirac field:

$$(\mathrm{i}\gamma^\mu \partial_\mu - m)\psi(x) = 0.$$

Derive the transformation rule for the Dirac field $\psi(x) \to \psi'(x')$ that is required in order for Dirac equation to be Lorentz-covariant.

(c) What is the conserved current due to the parity-invariance of the Dirac Lagrangian?

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Maximum marks: 3