FY3464 Quantum Field Theory
Problemset 11

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Problem 1
The term

$$i\bar{\psi}\gamma^5\psi \tag{1}$$

is Lorentz invariant. However, it violates CP symmetry. Prove that:

$$CP(i\bar{\psi}\gamma^5\psi)PC = -i\bar{\psi}\gamma^5\psi. \tag{2}$$

Problem 2
Consider a 0+1 fermionic harmonic oscillator. The path integral including source terms $\eta(t), \bar{\eta}(t)$ has the form

$$Z(\eta,\bar{\eta}) = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\Big(i\int dt[\bar{\psi}(t)(i\partial_0 - m_0 + i\varepsilon)\psi(t) + \bar{\eta}(t)\psi(t) + \bar{\psi}(t)\eta(t)]\Big). \tag{3}$$

Show in detail how it may be rewritten to

$$Z(\eta,\bar{\eta}) = Z(0,0)e^{-\int dt\bar{\eta}(t)S_F\eta(t)} \tag{4}$$

where $S_F(t-t')$ is the analog of the Dirac propagator in 3+1 dimensions, satisfying:

$$(i\partial_0 - m_0)S_F(t-t') = i\delta(t-t'), \tag{5}$$

and where we also defined

$$S_F\eta(t) \equiv \int dt' S_F(t-t')\eta(t'),\ \bar{\eta}S_F(t) = \int dt'\bar{\eta}(t')S_F(t'-t). \tag{6}$$

Hint: it may be useful to use the complete-square-and-shift-variables trick.

Problem 3
We saw in the lectures that if $m = 0$ in the Dirac Lagrangian, the left-handed and right-handed spinors decouple. While this is clear mathematically, can you think of a physical argument for why this decoupling occurs, while it is not possible for $m \neq 0$?