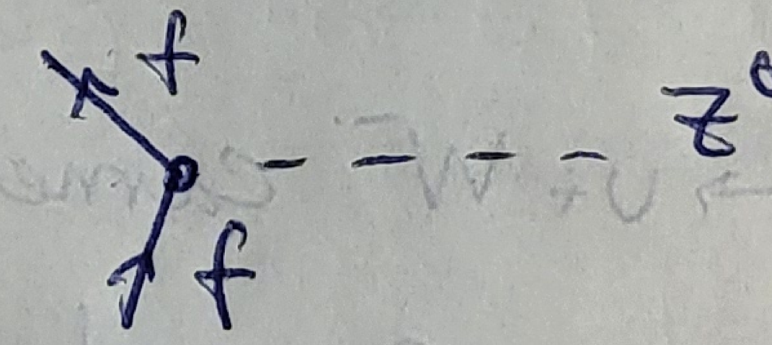
Not all entries are independent: U must be a unitary matrix. May write:

$$U = \begin{bmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{bmatrix}$$

where $c_i = \cos\theta_i$, $s_i = \sin\theta_i$. For $\theta_2 = \theta_3 = 0$, we recover the original Cabibbo-GIM picture with $\theta_1 = \theta_C$. Experimentally, one observes that θ_3 is finite, but small. δ is a phase which gives rise to CP-violation in weak int.

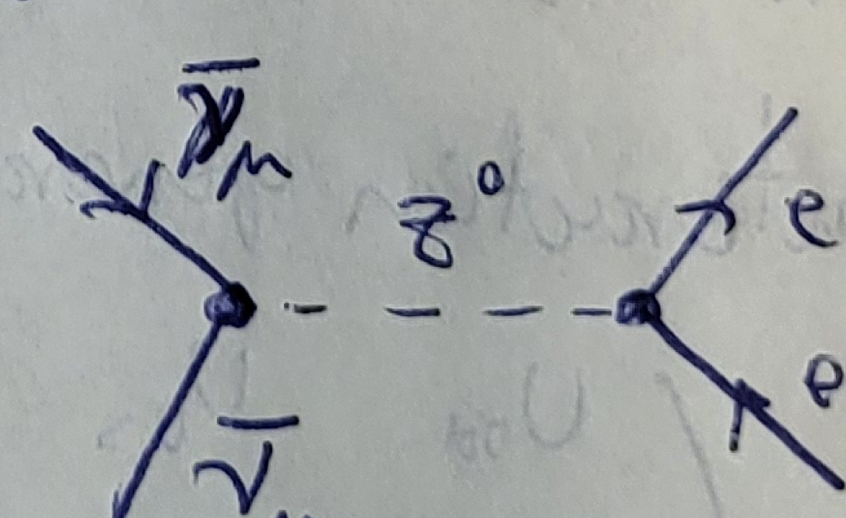
NEUTRAL WEAK INTERACTIONS

Fundamental vertex:



f = any lepton or quark (no fermion-mixing, though)

First experimental indication of neutral weak interaction in 1973: $\bar{\nu}_\mu + e \to \bar{\nu}_\mu + e$



Now, we know that the coupling of quarks and leptons to $W^\pm$ is of the form:

$$-\frac{i g_W}{2\sqrt{2}} \gamma^\mu (1-\gamma^5).$$

This is slightly modified when coupling to composite particles like the proton, but that's due to strong interaction contamination. The coupling to Z^0 is:

$$\frac{-ig_Z \gamma^\mu (c_V^f - c_A^f \gamma^5)}{2} \qquad (Z^0 \text{ vertex factor})$$

g_Z: neutral coupling constant, $c_{V,A}^f$: coefficients depending on f.

These numbers are determined by the weak mixing angle (Weinberg angle) θ_W. We have that:

$$g_W = \frac{g_e}{\sin\theta_W}$$

$$g_Z = \frac{g_e}{\sin\theta_W \cos\theta_W}$$

f	c_V	c_A
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	$\frac{1}{2}$
e^-, μ^-, τ^-	$-\frac{1}{2} + 2\sin^2\theta_W$	$-\frac{1}{2}$
u, c, t	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$
d, s, b	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$

This will be motivated later on (electroweak theory).

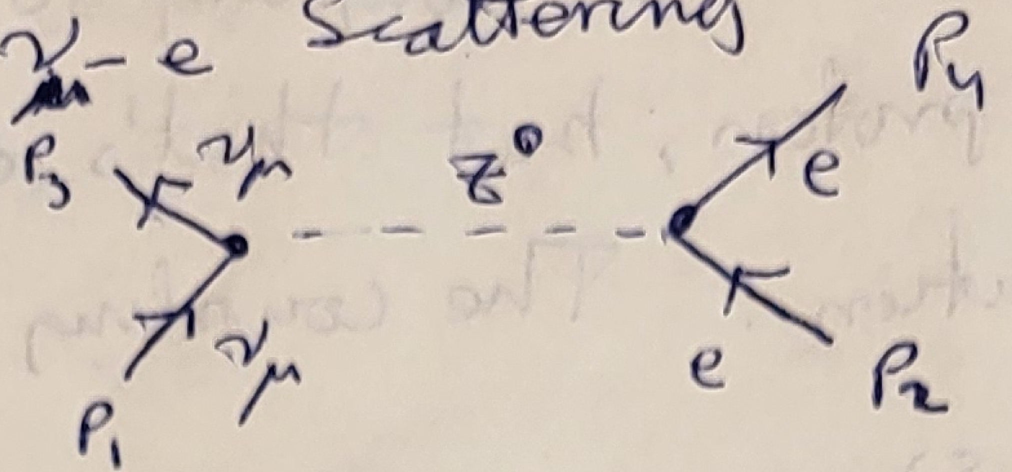
There is no way in the SM to calculate θ_W, but its value may be inferred from experiments: $\theta_W = 28.7^\circ$.

We already know the propagator:

$$\frac{-i(g_{\mu\nu} - q_\mu q_\nu / M_Z^2 c^2)}{q^2 - M_Z^2 c^2} \qquad \text{and} \qquad M_W = M_Z \cos\theta_W.$$

EXAMPLE Elastic $\nu_\mu - e$ Scattering

$\nu_\mu + e \rightarrow \nu_\mu + e$

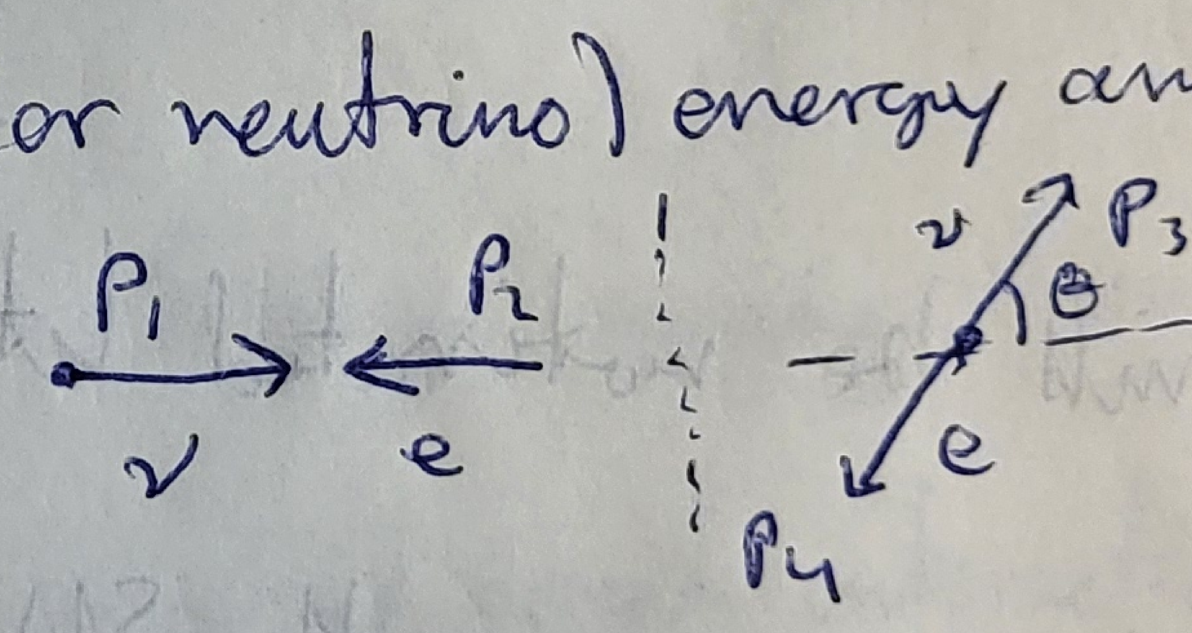


$$\mathcal{M} = \frac{g_z^2}{8(M_z c)^2}\left[\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)\right]\left[\bar{u}(4)\gamma_\mu(c_V - c_A\gamma^5)u(2)\right]$$

where $\{c_V, c_A\}$ are the neutral weak couplings for electrons. Go to the CM frame and assume very high energy scattering so that we may neglect to electron mass (rest energy). Then:

$$\langle|\mathcal{M}|^2\rangle = 2\left(\frac{g_z E}{M_z c^2}\right)^4\left[(c_V + c_A)^2 + (c_V - c_A)^2\cos^4\frac{\theta}{2}\right]$$

where E is the electron (or neutrino) energy and θ is the scattering angle



Diff. scatt. cross section for this situation was worked out in chapter 6:

$$\frac{d\sigma}{d\Omega} = 2\left(\frac{\hbar c}{\pi}\right)^2\left(\frac{g_z}{4M_z c}\right)^4 E^2\left[(c_V + c_A)^2 + (c_V - c_A)^2\cos^4\frac{\theta}{2}\right]$$

$$\Longrightarrow \sigma = \frac{2}{3\pi}(\hbar c)^2\left(\frac{g_z}{2M_z c^2}\right)^4 E^2\left(c_V^2 + c_A^2 + c_V c_A\right)$$

Note: most (not all) neutral processes are "masked" by competing EM ones. For instance, $e^+ + e^- \to \mu^+ + \mu^-$ can occur both via exchange of a virtual Z^0 or γ.

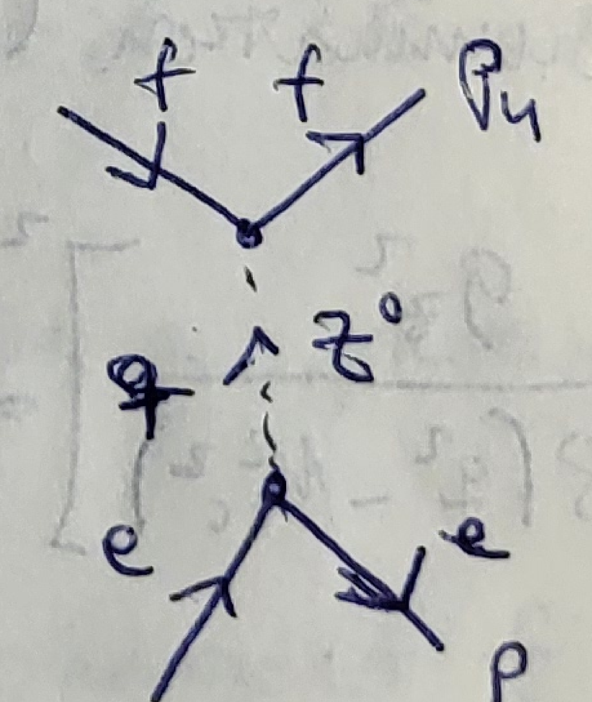
Moreover: there is a weak contamination in <u>every</u> EM process since Z^0 couples to everything that γ does (and more). Even if the effect is small, its smoking gun signature (when observable) is parity violation.

To access weak interactions alone, one can use neutrinos (with no EM coupling) or sufficiently high energies so that $q \sim M_Z c$ and hence the denominator of the Z^0 propagator is small $\to$ Large interaction.

EXAMPLE $e^- e^+$ - scattering near the Z^0 pole.

$e^+ + e^- \to f + \bar{f}$ where f is a quark or lepton.

Assume $m_f \ll M_Z$, but we use the exact form of the Z^0 propagator since we're interested in $q \sim M_Z c$.



By the way, note that Z^0 is its own antiparticle while W^+ and W^- are eachothers antiparticles.

Then, we get:

$$\mathcal{M} = -\frac{g_Z^2}{4[q^2-(M_Z c)^2]}\left[\bar{u}(4)\gamma^\mu(c_V^f - c_A^f\gamma^5)v(3)\right]$$

$$\times\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{(M_Z c)^2}\right)\left[\bar{v}(2)\gamma^\nu(c_V^e - c_A^e\gamma^5)u(1)\right]$$

with $q = p_1 + p_2 = p_3 + p_4$. Since we will consider energies near 90 GeV, we may safely neglect all masses.

From the second term, we find that q_μ contracts with γ^μ to give $\bar{u}(4)\,\not{q}\,(c_V - c_A\gamma^5)v(3)$.

Since $\not{q} = \not{p}_3 + \not{p}_4$ and $\bar{u}(4)\not{p}_4 = 0$ [Dirac eq. for $m=0$] in addition to:

$$\not{p}_3(c_V - c_A\gamma^5)v(3) = (c_V + c_A\gamma^5)\not{p}_3 v(3) = 0,$$

$q_\mu q_\nu$ then gives no contribution. Performing the spin summation (Casimir's trick):

$$\langle|\mathcal{M}|^2\rangle = \left[\frac{g_Z^2}{8(q^2 - M_Z^2c^2)}\right]^2 \mathrm{Tr}\left\{\gamma^\mu(c_V^f - c_A^f\gamma^5)\not{p}_3\gamma^\nu(c_V^f - c_A^f\gamma^5)\not{p}_4\right\}$$

$$\times\,\mathrm{Tr}\left\{\gamma_\mu(c_V^e - c_A^e\gamma^5)\not{p}_1\gamma_\nu(c_V^e - c_A^e\gamma^5)\not{p}_2\right\}$$

Performing the traces (using rules for γ-matrices) and integrating over the scattering angle, we find:

$$\sigma = \frac{1}{3\pi}\left[\frac{\hbar c g_Z^2 E}{4[(2E)^2-(M_Z c^2)^2]}\right]^2 [(c_V^f)^2+(c_A^f)^2][(c_V^e)^2+(c_A^e)^2]$$

in the CM frame.

When total energy $2E \to M_Z c^2$, σ diverges! To counter this (one would never observe a true mathematical divergence physically), take into account the finite lifetime τ_Z of Z^0:

$$\frac{1}{q^2-(M_Z c)^2} \longrightarrow \frac{1}{q^2-(M_Z c)^2 + i\hbar M_Z \Gamma_Z} \qquad (\Gamma_Z = \tau_Z^{-1}).$$

This "smears" out the mass, and leads to:

$$\sigma = \frac{(\hbar c g_Z^2 E)^2}{48\pi} \frac{[(c_V^f)^2+(c_A^f)^2][(c_V^e)^2+(c_A^e)^2]}{[(2E)^2-(M_Z c^2)^2]^2 + (\hbar M_Z c^2 \Gamma_Z)^2}$$

Correction may be neglected except if $2E \sim M_Z c^2$.

Now, the same process mediated by a photon gives:

$$\sigma = \frac{(\hbar c g_e^2)^2 (Q^f)^2}{48\pi E^2} \qquad (Q^f = \text{charge of } f \text{ in units of } e)$$

Compare γ- and Z^0- mediated scattering directly:

$$\frac{\sigma(e^+e^- \to Z^0 \to \mu^+\mu^-)}{\sigma(e^+e^- \to \gamma \to \mu^+\mu^-)} \simeq \frac{2E^4}{[(2E)^2-(M_Zc^2)^2]^2+(\hbar\Gamma_Z M_Zc^2)^2}$$

when inserting for Θ_W in $c_{V,A}^f$ and $c_{V,A}^e$.

$$\lim_{2E \ll M_Zc^2} \frac{\sigma_Z}{\sigma_\gamma} \simeq 2\left(\frac{E}{M_Zc^2}\right)^4 \ll 1$$

$$\lim_{2E \to M_Zc^2} \frac{\sigma_Z}{\sigma_\gamma} \simeq \frac{1}{8}\left(\frac{M_Zc^2}{\hbar\Gamma_Z}\right)^2 \gg 1$$

Have used $\hbar\Gamma_Z = 2.5$ GeV. Hence, weak mechanism strongly favored near Z^0 pole.

ELECTROWEAK UNIFICATION

Chiral fermion states

We would now like to explore where the GWS parameters $[c_V, c_A = c_V(\Theta_W), c_A(\Theta_W)]$ and $g_W, g_Z, M_W = M_Z \cos\Theta_W$ all come from.

First, note that Glashow's original aim was to unify weak and EM interactions as manifestations of one fundamental "electroweak" interaction.

Problem: if it's the same underlying interaction, why is γ massless and $W^\pm$ & Z^0 so heavy?

Solution: Higgs mechanism (next chapter).

What about structural difference between weak and EM vertices: γ^μ vs. $\gamma^\mu(1-\gamma^5)$ for $W^\pm$ (maximal mixing of vector-axially)?

This can be fixed by absorbing $(1-\gamma^5)$ into the particle spinor itself:

$$u_L(p) \equiv \frac{(1-\gamma^5)}{2} u(p).$$

In general, u_L is not an eigenstate of helicity operator in spite of "L" representing "left-handed", i.e. helicity -1:

$$\gamma^5 u(p) = \begin{pmatrix} \frac{c(\vec{p}\cdot\vec{\sigma})}{E+mc^2} & 0 \\ 0 & \frac{c(\vec{p}\cdot\vec{\sigma})}{E-mc^2} \end{pmatrix} u(p) \qquad (\text{not} \propto u(p))$$

PROOF: $\gamma^5 u = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} u_A \\ u_B \end{pmatrix} = \begin{pmatrix} u_B \\ u_A \end{pmatrix}$. Now use that $(\not{p} - mc)u = 0 = \begin{pmatrix} \left(\frac{E}{c} - mc\right)u_A - \vec{p}\cdot\vec{\sigma}\, u_B \\ \vec{p}\cdot\vec{\sigma}\, u_A - \left(\frac{E}{c} + mc\right)u_B \end{pmatrix}$

$\Rightarrow u_A = \frac{c}{E-mc^2} \cdot \vec{p}\cdot\vec{\sigma}\, u_B$ & $u_B = \frac{c}{E+mc^2}(\vec{p}\cdot\vec{\sigma})u_A$. Insert above and get

$$\gamma^5 u = \begin{pmatrix} \frac{c}{E+mc^2}\vec{p}\cdot\vec{\sigma} & \\ & \frac{c}{E-mc^2}\vec{p}\cdot\vec{\sigma} \end{pmatrix} u. \quad \text{qed.}$$

If $m=0$, then $\gamma^5 u(p) = (\hat{p}\cdot\underline{\Sigma})u(p)$ where $\hat{p}\cdot\underline{\Sigma}$ is the helicity operator with eigen values ± 1.

$$\frac{1}{2}(1-\gamma^5)u(p) = \begin{cases} 0 & \text{if } u(p) \text{ has helicity } +1 \\ u(p) & \text{if } u(p) \text{ has helicity } -1. \end{cases} \quad (\text{for } m=0)$$

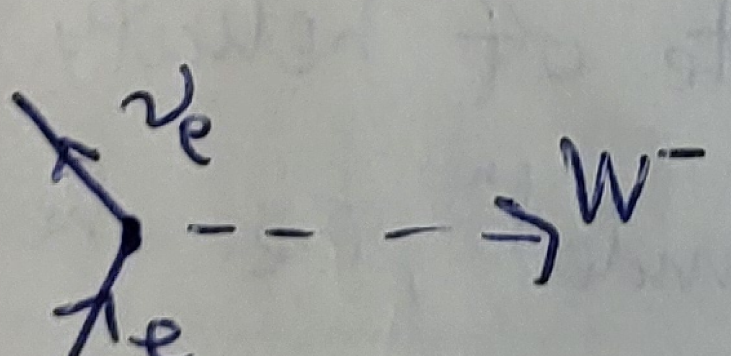
This holds exactly for $m=0$ only, but u_L is always called a left-handed particle: the projection operator $\frac{1}{2}(1-\gamma^5)$ picks out the -1 helicity component from $u(p)$. [for $m=0$]

For an anti particle: $v_L(p) \equiv \frac{(1+\gamma^5)}{2} v(p)$.

For right-handed counterpart, let $\gamma^5 \to (-\gamma^5)$.

$\Rightarrow$ Chiral fermion states ("chiral" is Greek for hand)

Now: weak & EM interactions can be expressed in a more unified form.

Consider  which contributes

$$j_\mu^- = \bar{\nu}\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)e \text{ to } \mathcal{M} \quad [\bar{\nu} \text{ and } e \text{ represent the spinors}].$$

j_μ^-: weak current (analogue to electric current in QED)