MORE ON NEUTRINO OSCILLATIONS

There was a problem associated with the detected neutrinos, however: only saw about 1/3 of what theory predicted that the Sun should produce.

$\Rightarrow$ The solar neutrino problem.

First, one was willing to attribute this to the experimental setup (33 events from a 615 ton tank). But other experiments using different techniques confirmed the deficit.

Pontecorvo suggested in 1968 an elegant solution to the problem: what if the ν_e produced by the Sun were transformed into something else on their way to Earth which could not be detected by these experiments, such as ν_μ?

$\Rightarrow$ Neutrino oscillations.

To illustrate the idea, consider a scenario where we have two neutrino flavors ν_e and ν_μ. If indeed one may spontaneously convert to another, neither can be an eigenfunction of the Hamiltonian. Instead, the stationary states are some linear combination

$$\nu_1(t) = \cos\theta\, \nu_\mu(t) - \sin\theta\, \nu_e(t) \quad , \quad \nu_2(t) = \sin\theta\, \nu_\mu(t) + \cos\theta\, \nu_e(t) \qquad (*)$$

This form ensures orthonormality of ν_1 and ν_2.

These states have a simple time dependence

$$\nu_i(t) = \nu_i(0)\, e^{-iE_i t/\hbar} \quad \text{where } E_i = \sqrt{p_i^2 c^2 + m_i^2 c^4} \text{ is}$$

their energy and m_i their mass. We note immediately that it follows that ν_e and ν_μ do not have well-defined masses since they are lin. comb. in $(*)$

To see how a flavor state evolves with time (e.g. from production in sun and propagating toward Earth), assume we start with a ν_e:

$\nu_e(t=0) = 1, \ \nu_\mu(t=0) = 0 \Rightarrow \nu_1(0) = -\sin\theta, \ \nu_2(0) = \cos\theta.$

Now rewrite (*) to express $\{\nu_e, \nu_\mu\}$ as function of $\{\nu_1, \nu_2\}$ to obtain

$$\nu_\mu(t) = \cos\theta\, \nu_1(t) + \sin\theta\, \nu_2(t) = \sin\theta\cos\theta \left(e^{-iE_1t/\hbar} + e^{-iE_2t/\hbar}\right).$$

We then find the probability that ν_e has converted into ν_μ after a time t:

$$|\nu_\mu(t)|^2 = \frac{\sin^2 2\theta}{4}\left(1 - e^{i(E_2-E_1)t/\hbar} - e^{-i(E_2-E_1)t/\hbar} + 1\right)$$

$$= \frac{\sin^2 2\theta}{4} \cdot 4\sin^2\left(\frac{E_2-E_1}{2\hbar}t\right)$$

$$\Rightarrow P_{\nu_e \to \nu_\mu} = \left[\sin 2\theta \sin\left(\frac{E_2-E_1}{2\hbar}t\right)\right]^2$$

In this model, we thus have precisely a conversion between ν_e and ν_μ which depends in an oscillatory manner on t (or distance travelled).

There are two necessary ingredients for neutrino oscillations to occur:

1) Mixing $\theta \neq 0$
2) Different masses for the eigenstates (both cannot be zero).

There is thus experimental evidence today that neutrinos are not massless and there is no fundamental reason for why they should be (unlike e.g. the photon). In practical calculations where ν-oscillations are not of relevance, we will often approximate their mass to zero which yields very good results.