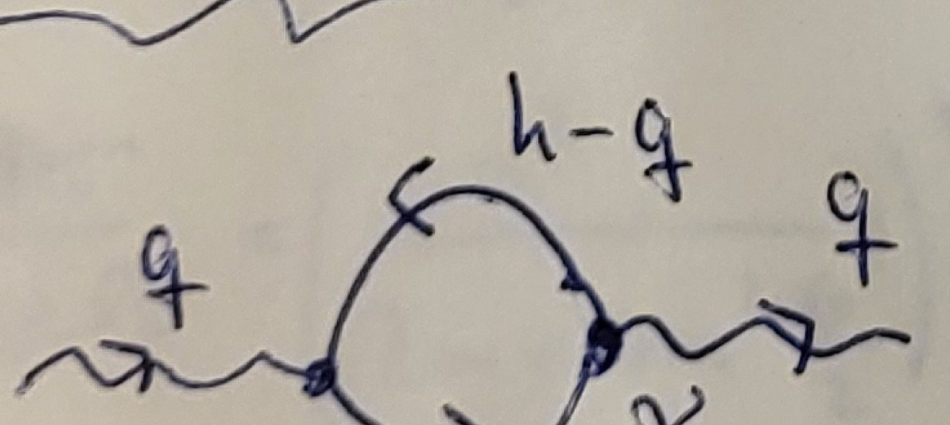
$\boxed{\text{INTERLUDE}}$: treatment of bubble diagrams (fermion loops).

The general rule is:

- Include a factor (-1), take the Tr, and follow the fermion lines along the arrows (opposite for external lines).

The Tr corresponds to including all possible spin orientations of the fermion as it connects with itself in the end (just like integration over momenta).

Example



$$\Rightarrow -\mathrm{Tr}\int \frac{d^4k}{(2\pi)^4}\,\frac{\{\gamma_\mu(\not k-\not q+m)\gamma_\nu(\not k+m)\}}{[(k-q)^2-m^2c^2][k^2-m^2c^2]}$$

But is the direction important? Could we also go in the opposite direction, against the arrows?

The answer is that for a loop with 2 (even) number of fermions, we can. The proof is:

$$\mathrm{Tr}\{\gamma_\mu \not{p} \gamma_\nu \not{q}\} = p^\lambda q^\sigma \,\mathrm{Tr}\{\gamma_\mu \gamma_\lambda \gamma_\nu \gamma_\sigma\}$$

$$= 4 p^\lambda q^\sigma \left(g_{\mu\lambda} g_{\nu\sigma} - g_{\mu\nu} g_{\lambda\sigma} + g_{\mu\sigma} g_{\lambda\nu}\right)$$

$$= 4\left(p_\mu q_\nu - g_{\mu\nu} p^\lambda q_\lambda + q_\mu p_\nu\right) \qquad (*)$$

Now the Tr if we go in the opposite direction is: $\mathrm{Tr}\{\gamma_\mu \not{q} \gamma_\nu \not{p}\}$. But we see that (*) is invariant when exchanging $p \leftrightarrow q$. Therefore, the two traces are the same. □

In fact, this is a special case of the more general statement:

$$\mathrm{Tr}\{\gamma_{\mu_1} \gamma_{\mu_2} \cdots \gamma_{\mu_n}\} = \mathrm{Tr}\{\gamma_{\mu_n} \cdots \gamma_{\mu_2} \gamma_{\mu_1}\}.$$