FY3403 Particle physics
Problemset 5 fall 2024

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Problem 1. Isospin classification
Write down the isospin classification $|II_3\rangle$ of the following elementary particles:
(i) Ω^-, (ii) Σ^+, (iii) Ξ^0, (iv) Δ^0, (v) ρ^+, (vi) η, (vii) $\bar{K}^0$.

Problem 2. Isospin analysis of pion-nucleon scattering
In this problem you should use the isospin symmetry of strong interactions to find relations between the cross-sections for the following processes

1. $\pi^+ + p \longrightarrow \pi^+ + p$
2. $\pi^0 + p \longrightarrow \pi^0 + p$
3. $\pi^- + p \longrightarrow \pi^- + p$
4. $\pi^+ + n \longrightarrow \pi^+ + n$
5. $\pi^0 + n \longrightarrow \pi^0 + n$
6. $\pi^- + n \longrightarrow \pi^- + n$
7. $\pi^+ + n \longrightarrow \pi^0 + p$
8. $\pi^0 + p \longrightarrow \pi^+ + n$
9. $\pi^0 + n \longrightarrow \pi^- + p$
10. $\pi^- + p \longrightarrow \pi^0 + n$

a) Write the isospin contents of the states involved (on the form $|I^{(1)}I_3^{(1)}\rangle\,|I^{(2)}I_3^{(2)}\rangle$), and decompose these into a sum over states with fixed isospin (on the form $|II_3\rangle$), for each of the cases above.

b) The assumption of isospin symmetry means that the scattering amplitude $\mathcal{M}$ only depends on the total isospin,

$$\langle II_3|S|I'I_3'\rangle = \mathcal{M}^{(I)}\,\delta_{II'}\,\delta_{I_3I_3'}, \tag{1}$$

where the amplitudes $\mathcal{M}^{(I)}$ generally are complex numbers. They give information about how likely it is that an initial state (before the scattering) has turned into a specific final state (after the scattering). Here, S denotes the S-matrix that determines how an initial state $|i\rangle$ has evolved with time, i.e. $|\Phi(t=\infty)\rangle = S|i\rangle$. In turn, S is determined by the way the particles involved in the scattering process interact with each other. For instance, we would then have

$$\langle\frac{3}{2},\frac{1}{2}\Big|S\Big|\frac{3}{2},\frac{1}{2}\rangle = \mathcal{M}^{(3/2)}, \tag{2}$$

while

$$\langle \frac{3}{2}, \frac{1}{2} \Big| S \Big| \frac{1}{2}, \frac{1}{2} \rangle = 0. \tag{3}$$

Express the alltogether ten scattering amplitudes $\mathcal{M}_i$ for the above processes in terms of the amplitudes $\mathcal{M}^{(I)}$.

c) The scattering cross-sections σ_i for the processes above has the form $\sigma_i = C\,|\mathcal{M}_i|^2$, where C is the same for all processes. Find all possible relations (that can be derived from isospin symmetry) between these cross-sections.

d) When the center-of-mass energy E is at the Δ–resonance, $E \approx 1232$ MeV, the amplitude $\mathcal{M}^{(3/2)}$ becomes completely dominating. Which relations between cross-sections can be deduced in this case?