NTNU
Institutt for fysikk

FY3403 Particle physics
Problemset 4 fall 2024

Problem 1. The symmetry group of a square

In the lectures we studied the symmetry group of an equilatrial triangle. In this problem you should repeat this for a square.

a) How many elements are there in this group?

b) Find the multiplication table for this group.

c) Is this group abelian or non-abelian?

d) Find all subgroups of this group.

Problem 2. Rotation of spin-$\frac{1}{2}$ states

The rotation operator $R(\theta, \hat{n})$ for spin-$\frac{1}{2}$ wave functions (spinors) $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$ is

$$R(\theta, \hat{n}) = \mathrm{e}^{\mathrm{i}\theta\, \hat{n}\cdot\boldsymbol{\sigma}/2}, \tag{1}$$

where θ is the angle rotated and $\hat{n}$ is the direction of the rotation axis.

a) Explain geometrically why we should have $R(-\theta, -\hat{n}) = R(\theta, \hat{n})$.

b) Show, f.i. by series expansion of the exponential, that

$$R(\theta, \hat{n}) = \cos(\theta/2) + \mathrm{i}\sin(\theta/2)\, \hat{n}\cdot\boldsymbol{\sigma}. \tag{2}$$

c) For a state with spin along the z-axis the corresponding spinor is $\chi = \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Rotate this state an angle $-\theta$ about the y-axis, and show that the rotated spinor becomes $\chi_\theta = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix}$.

d) Show that χ_θ is an eigenstate of $\sigma_{\hat{n}} \equiv \hat{n}\cdot\boldsymbol{\sigma}$, where $\hat{n}$ is the vector obtained by rotating the unit vector $\hat{e}_z$ an angle θ about the y-axis. Draw a figure.