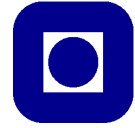


**FY3403 Particle physics**  
**Problemset 4 fall 2024**

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**Problem 1. The symmetry group of a square**

In the lectures we studied the symmetry group of an equilateral triangle. In this problem you should repeat this for a square.

- a) How many elements are there in this group?
- b) Find the multiplication table for this group.
- c) Is this group abelian or non-abelian?
- d) Find all subgroups of this group.

**Problem 2. Rotation of spin- $\frac{1}{2}$  states**

The rotation operator  $R(\theta, \hat{n})$  for spin- $\frac{1}{2}$  wave functions (spinors)  $\chi = \begin{pmatrix} a \\ b \end{pmatrix}$  is

$$R(\theta, \hat{n}) = e^{i\theta \hat{n} \cdot \boldsymbol{\sigma}/2}, \quad (1)$$

where  $\theta$  is the angle rotated and  $\hat{n}$  is the direction of the rotation axis.

- a) Explain geometrically why we should have  $R(-\theta, -\hat{n}) = R(\theta, \hat{n})$ .
- b) Show, f.i. by series expansion of the exponential, that

$$R(\theta, \hat{n}) = \cos(\theta/2) + i \sin(\theta/2) \hat{n} \cdot \boldsymbol{\sigma}. \quad (2)$$

- c) For a state with spin along the  $z$ -axis the corresponding spinor is  $\chi = \chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ .

Rotate this state an angle  $-\theta$  about the  $y$ -axis, and show that the rotated spinor becomes

$$\chi_\theta = \begin{pmatrix} \cos \theta/2 \\ \sin \theta/2 \end{pmatrix}.$$

- d) Show that  $\chi_\theta$  is an eigenstate of  $\sigma_{\hat{n}} \equiv \hat{n} \cdot \boldsymbol{\sigma}$ , where  $\hat{n}$  is the vector obtained by rotating the unit vector  $\hat{e}_z$  an angle  $\theta$  about the  $y$ -axis. Draw a figure.