

FY3403 Particle physics
Problemset 3 fall 2024

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Problem 1. Kinematics of Compton scattering

In an experiment carried out by A.H. Compton in 1923 it was demonstrated that light scattered from a charged particle at rest will change its wavelength according to the formula

$$\lambda' = \lambda + \frac{h}{mc} (1 - \cos \theta) \quad (1)$$

where λ is the wavelength of the incoming light, λ' the wavelength of the scattered light, θ the scattering angle, and m the mass of the particle. This can be viewed as the definite proof of the particle nature of the photon.

- a) Consider the incoming light as a massless particle with momentum $p = \frac{h}{\lambda}$ and energy $E_\gamma = pc$ (where h is the Planck constant), and write down the conservation laws for energy and momentum for the scattering process.
- b) Solve the conservation laws with respect to the momentum p' of the scattered photon.
- c) Show that the result from the previous point is in accordance with equation (1).

Hint: A pedestrian way to attack the problem is¹ to first eliminate $p_y^{(m)}$, then $p_x^{(m)}$ (where $p^{(m)}$ is the momentum of the particle after the scattering process), and finally solve for p' (the absolute value of the photon momentum after the scattering process).

Given: The relativistic connection between energy and momentum for a particle with mass m is

$$E = \sqrt{(pc)^2 + (mc^2)^2}. \quad (2)$$

Problem 2. Unstable particles produced by cosmic rays

Cosmic rays will produce muons, $\mu^\pm$, high in the atmosphere, assume at a height of 8 km. Assume that the muons have a kinetic energy T and a velocity directed towards the center of the earth. What is the probability that a muon will reach the surface of the earth if

- a) $T = 20$ MeV
- b) $T = 20$ GeV
- c) Cosmic rays will also produce pions, e.g. $\pi^\pm$, at the same height. Repeat the calculation for such particles

In all these calculations you may neglect interactions between the produced particles and the atmosphere.

Given:

$$m_\mu = 105.66 \text{ MeV}, \tau_\mu = 2.197 \times 10^{-6} \text{ s}$$

$$m_{\pi^\pm} = 139.57 \text{ MeV}, \tau_{\pi^\pm} = 2.603 \times 10^{-8} \text{ s}$$

¹Assume the incoming photon is moving in the positive x -direction and that the scattering process happens in the xy -plane.

Problem 3. Kinematics of π^0 -decay

A neutral pion may decay into two photons

$$\pi^0 \rightarrow \gamma_1 + \gamma_2 \quad (3)$$

- a) Use conservation of four-momentum to find the energy of the two photons (measured in MeV) when the pion is at rest before the decay.
- b) Next assume that the pion has a momentum $p_\pi = 100 \text{ MeV}/c$ when it decays. One of the photons emerge at an angle $\theta_1 = 60^\circ$ relative to $\mathbf{p}_\pi$. Find the energy of this photon. Also find the energy and direction of motion of the other photon.
- c) Neutral pions with known energy E_π may decay into two photons. In each decay we observe the two photons and measure the angle θ_{12} between them. Show that we may determine the mass m_{π^0} of the pion if we know the energy E_π , and have found the least possible value $\theta_{\min}$ of the opening angle θ_{12} .