FY3403 Particle physics
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Problem 1. Neutrino oscillations

When an electron antineutrino is created in a radioactive process like β-decay,

$$n \longrightarrow p + e + \bar{\nu}_e, \tag{1}$$

some might argue that the antineutrino is not a genuine particle in the sense that it does not have a well-defined mass. To simplify mathematics we consider only two generations. Then the states of $\bar{\nu}_e$ and $\bar{\nu}_\mu$ are linear, orthonormal superpositions of two antineutrino states with definite mass,

$$\begin{pmatrix} |\,\bar{\nu}_e\,\rangle \\ |\,\bar{\nu}_\mu\,\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\,\bar{\nu}_1\,\rangle \\ |\,\bar{\nu}_2\,\rangle \end{pmatrix}, \tag{2}$$

where the antineutrinos $\bar{\nu}_1$ and $\bar{\nu}_2$ have (small) masses m_1 and m_2.

a) Assume now that $\bar{\nu}_e$ is created with a fixed momentum $\boldsymbol{p}$ (such that $|\boldsymbol{p}| \gg m_1 c,\, m_2 c$) at time $t = 0$, and denote this state by $|\,\Psi_{\boldsymbol{p}}(0)\rangle = |\,\bar{\nu}_e\,\rangle$. Find $|\,\Psi_{\boldsymbol{p}}(t)\rangle$ expressed by $|\,\bar{\nu}_1\,\rangle$, $|\,\bar{\nu}_2\,\rangle$ and the parameters θ, m_1 and m_2.

b) After time t the neutrino has moved a distance $L = ct$ (it moves very closely to the speed of light, since $|\boldsymbol{p}| \gg m_1 c,\, m_2 c$). Assume there is a neutrino detector at that distance from the creation point, based on a process of inverse β-decay,

$$\bar{\nu}_e + X \longrightarrow e^+ + Y \tag{3}$$

where the nucleus Y is related to the nucleus X by the transformation of one proton into a neutron. To evaluate the probability for this one must express the state $|\,\Psi_{\boldsymbol{p}}(t)\rangle$ by the states $|\,\bar{\nu}_e\rangle$ and $|\,\bar{\nu}_\mu\rangle$. Do that to find the expression for the expansion coefficients $c_{\bar{e}}(t)$ and $c_{\bar{\mu}}(t)$. You may simplify the result by using the fact that

$$\sqrt{\boldsymbol{p}^2 c^2 + m_i^2 c^4} \approx |\boldsymbol{p}|c + \frac{m_i^2 c^3}{2|\boldsymbol{p}|}.$$

c) Show that the probability of finding an electron antineutrion, $p_{\bar{e}}(t) = |c_{\bar{e}}(t)|^2$, is a simple oscillating function of L, and find an expression for the oscillation length L_0 (wavelength of oscillation).

Problem 2. The particle content of the Standard Model

a) List all the "building blocks" of matter particles in *the Standard Model for Elementary Particles*.

b) How are (i) *baryons*, (ii) *mesons*, (iii) *hadrons*, and (iv) *leptons* made out of these building blocks (to first approximation)?

c) According to the classification above, of which particle type are the following elementary particles: (i) π^+, (ii) Δ^-, (iii) n, (iv) τ, (v) $\bar{\nu}_\mu$, (vi) K^+, (vii) Ω^-, (viii) Ξ^+?

d) List all "messenger" particles in the Standard Model, i.e. those which carry (the known) fundamental forces.

Problem 3. Processes in the Standard Model

Which of the processes below to you believe is possible according to the Standard Model? Which type of interaction is involved if the process is possible? Which conservation law is involved if the process is impossible?

1. $p + \bar{p} \to \pi^+ + \pi^0$
2. $e^+ + e^- \to \mu^+ + \mu^-$
3. $e^+ \to \mu^+ + \nu_\mu + \bar{\nu}_e$
4. $\Sigma^- \to n + \pi^-$
5. $\mu^- \to e^- + \bar{\nu}_e$
6. $\bar{\nu}_e + p \to n + e^+$
7. $e + p \to \nu_e + \pi^0$
8. $\pi^0 \to \gamma + \gamma$
9. $\pi^0 \to \gamma + \gamma + \gamma$
10. $\pi^- \to \mu^- + \bar{\nu}_\mu$
11. $\pi^- \to e^- + \bar{\nu}_e$