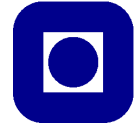


FY3403 Particle physics
Problemset 11 fall 2024

PAGE 1 OF 2

NTNU



Institutt for
fysikk

Background material: mainly Griffiths chapter 10.

Problem 1. Spontaneous symmetry breaking (SSB)

This exercise is meant to illustrate the important mechanism of *spontaneous symmetry breaking*, where a symmetric potential can lead to ground states where the symmetry is in some sense 'lost'.

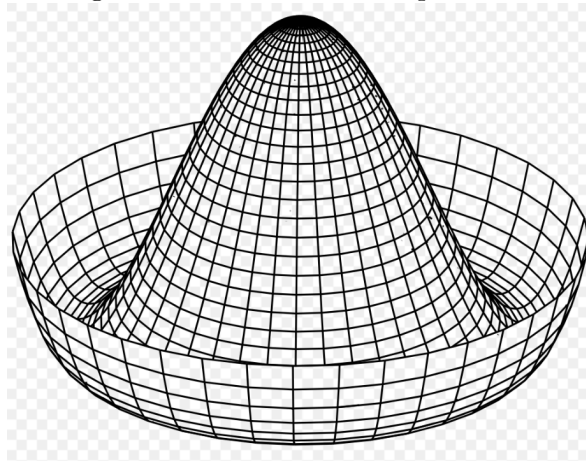
Consider a classical particle parameterized by the two-dimensional coordinate $r = (x(t), y(t))$, and let it travel according to the following Hamiltonian

$$H = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - V(x, y) \quad (1)$$

where $V(x, y) = a(x^2 + y^2) + b(x^2 + y^2)^2 = a(x^2 + y^2) + b(x^4 + 2x^2y^2 + y^4)$

- What symmetries does the potential possess?
- Consider the case $a, b > 0$. What is the minimum of the potential? At which point does this minimum occur?
- Suppose a particle is situated at the minimum (x_{min}, y_{min}) . A way to check if this minimum is stable is to change to polar coordinates centered at (x_{min}, y_{min}) and calculate how the potential looks like locally around $r = 0$. Taylor expand $V(x, y)$ around (x_{min}, y_{min}) to second order and express the result in polar coordinates. Argue whether the minima are stable or not.
- Consider now the case $a < 0 < b$. What is the minimal value now? At which point(s) does this minimal value occur? (*Hint:* the potential is drawn for this case in Fig. 1.)

Figure 1: The Mexican hat potential.



- e) Pick one of the points where the minimal value occur and do the Taylor expansion as above. Argue whether your minima is stable or not. (*Hint:* remember to use polar coordinates centered at the point you chose.)
- f) If you have done all the above exercises correctly, you will have observed that the ground state in c is stable and retains the original symmetry of the potential, whereas the ground state(s) in d does not. At what point in your calculation in d do you actually lose the symmetry of the initial problem?

Problem 2. Mass generation from SSB

A full treatment of the Higgs mechanism in the Standard model requires writing down a formidable Lagrangian taking into account interactions between many different fields. This exercise is instead meant to show the basic mechanism by which mass terms can be modified in a Lagrangian when symmetries are broken.

Consider the Lagrangian

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 - V(x), V(x) = -ax^2 + bx^4 \quad (2)$$

where $a, b > 0$

- a) Complete the square in the potential term and discard any constants.
- b) Introduce the variable $y = x^2 - \frac{a}{2b}$ and write $\mathcal{L}$ as a function of y .
- c) Assume y is small and Taylor expand to leading order. What is the mass term in this new Lagrangian?