

FY3403 Particle physics
Problemset 9

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SUGGESTED SOLUTION

Problem 8.16

(a) Here

$$c_1 = c_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad c_2 = c_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

From Eq. 8.47,

$$f = \frac{1}{4} \left[(0 \ 1 \ 0) \lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] \left[(0 \ 0 \ 1) \lambda^\alpha \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right] = \frac{1}{4} \lambda_{22}^\alpha \lambda_{33}^\alpha.$$

The only λ matrix with a nonzero entry in the 33 position is λ^8 , so

$$f = \frac{1}{4} \lambda_{22}^8 \lambda_{33}^8 = \frac{1}{4} \left(\frac{1}{\sqrt{3}} \right) \left(-\frac{2}{\sqrt{3}} \right) = -\frac{1}{6}. \quad \checkmark$$

(b)

$$c_1 = c_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \text{ for the first term, } c_1 = c_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ for the second :}$$

$$f = \frac{1}{4} \frac{1}{\sqrt{2}} \left\{ \left[c_3^\dagger \lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] [(1 \ 0 \ 0) \lambda^\alpha c_4] - \left[c_3^\dagger \lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] [(0 \ 1 \ 0) \lambda^\alpha c_4] \right\}$$

The outgoing quarks are in the same color state $(r\bar{r} - b\bar{b})/\sqrt{2}$, so

$$c_3 = c_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \text{ for the first term, } c_3 = c_4 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ for the second.}$$

There are 4 terms in all:

$$\begin{aligned} f = \frac{1}{4} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} & \left\{ \left[(1 \ 0 \ 0) \lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \left[(1 \ 0 \ 0) \lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \right. \\ & - \left[(0 \ 1 \ 0) \lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \left[(0 \ 1 \ 0) \lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \\ & - \left[(1 \ 0 \ 0) \lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] \left[(0 \ 1 \ 0) \lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] \\ & \left. + \left[(0 \ 1 \ 0) \lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] \left[(0 \ 1 \ 0) \lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] \right\} \end{aligned}$$

$$\begin{aligned}
f &= \frac{1}{8} (\lambda_{11}^\alpha \lambda_{11}^\alpha - \lambda_{21}^\alpha \lambda_{12}^\alpha - \lambda_{12}^\alpha \lambda_{21}^\alpha + \lambda_{22}^\alpha \lambda_{22}^\alpha) = \frac{1}{8} \left[(\lambda_{11}^3 \lambda_{11}^3 + \lambda_{11}^8 \lambda_{11}^8) \right. \\
&\quad \left. - (\lambda_{21}^1 \lambda_{12}^1 + \lambda_{21}^2 \lambda_{12}^2) - (\lambda_{12}^1 \lambda_{21}^1 + \lambda_{12}^2 \lambda_{21}^2) + (\lambda_{22}^3 \lambda_{22}^3 + \lambda_{22}^8 \lambda_{22}^8) \right] \\
&= \frac{1}{8} \left\{ [(-1)(-1) + \left(\frac{1}{\sqrt{3}}\right) \left(\frac{1}{\sqrt{3}}\right)] - [(1)(1) + (i)(-i)] \right. \\
&\quad \left. - [(1)(1) + (-i)(i)] + \left[(-1)(-1) + \left(\frac{1}{\sqrt{3}}\right) \left(\frac{1}{\sqrt{3}}\right)\right] \right\} \\
&= \frac{1}{8} \left(1 + \frac{1}{3} - 1 - 1 - 1 - 1 + 1 + \frac{1}{3} \right) = \frac{1}{8} \left(-\frac{4}{3} \right) = -\frac{1}{6}. \quad \checkmark
\end{aligned}$$

(c)

$$\begin{aligned}
f &= \frac{1}{4} \frac{1}{\sqrt{6}} \left\{ \left[c_3^\dagger \lambda^\alpha \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right] [(1 \ 0 \ 0) \lambda^\alpha c_4] + \left[c_3^\dagger \lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right] [(0 \ 1 \ 0) \lambda^\alpha c_4] \right. \\
&\quad \left. - 2 \left[c_3^\dagger \lambda^\alpha \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right] [(0 \ 0 \ 1) \lambda^\alpha c_4] \right\}
\end{aligned}$$

There are nine terms,

$$\begin{aligned}
f &= \frac{1}{4} \frac{1}{\sqrt{6}} \frac{1}{\sqrt{6}} [(\lambda_{11}^\alpha \lambda_{11}^\alpha + \lambda_{21}^\alpha \lambda_{12}^\alpha - 2\lambda_{31}^\alpha \lambda_{13}^\alpha) + (\lambda_{12}^\alpha \lambda_{21}^\alpha + \lambda_{22}^\alpha \lambda_{22}^\alpha - 2\lambda_{32}^\alpha \lambda_{23}^\alpha) \\
&\quad - 2(\lambda_{13}^\alpha \lambda_{31}^\alpha + \lambda_{23}^\alpha \lambda_{32}^\alpha - 2\lambda_{33}^\alpha \lambda_{33}^\alpha)] \\
&= \frac{1}{24} \left\{ \left(1 + \frac{1}{3}\right) + (1 + 1) - 2(1 + 1) + (1 + 1) + \left(1 + \frac{1}{3}\right) - 2(1 + 1) \right. \\
&\quad \left. - 2 \left[(1 + 1) + (1 + 1) - 2 \left(\frac{4}{3}\right) \right] \right\} \\
&= \frac{1}{24} \left(\frac{4}{3} + 2 - 4 + 2 + \frac{4}{3} - 4 - 4 - 4 + \frac{16}{3} \right) = \frac{1}{24} (-4) = -\frac{1}{6}. \quad \checkmark
\end{aligned}$$

Problem 8.23

From Eq. 8.90,

$$\Gamma(\eta_c \rightarrow 2g) = \frac{8\pi}{3c} \left(\frac{\hbar\alpha_s}{m} \right)^2 |\psi(0)|^2.$$

For the decay of positronium we had (Eq. 7.168 and 7.171)

$$\sigma = \frac{4\pi}{c v} \left(\frac{\hbar\alpha}{m} \right)^2 \Rightarrow \Gamma = \frac{4\pi}{c} \left(\frac{\hbar\alpha}{m} \right)^2 |\psi(0)|^2.$$

In the case of η_c the electron/positron are replaced by $c/\bar{c}$, so the charge $-e \rightarrow \frac{2}{3}e$, and hence $\alpha \rightarrow \frac{4}{9}\alpha$. Meanwhile, the amplitude picks up a factor of $1/\sqrt{3}$ (from Eq. 8.30) for each of the three $q\bar{q}$ pairs, and there are three of these, so the color factor in the amplitude is $3/\sqrt{3} = \sqrt{3}$, so Eq. 7.168 is multiplied by 3—and Eq. 7.171 inherits this factor:

$$\Gamma(\eta_c \rightarrow 2\gamma) = 3 \left(\frac{4}{9} \right)^2 \frac{4\pi}{c} \left(\frac{\hbar\alpha}{m} \right)^2 |\psi(0)|^2 = \left(\frac{8}{9} \right) \frac{4\pi}{c} \left(\frac{\hbar\alpha}{m} \right)^2 |\psi(0)|^2.$$

Thus

$$\frac{\Gamma(\eta_c \rightarrow 2g)}{\Gamma(\eta_c \rightarrow 2\gamma)} = \boxed{\left(\frac{9}{8} \right) \left(\frac{\alpha_s}{\alpha} \right)^2}.$$