FY3403 Particle physics

Problemset 7

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SUGGESTED SOLUTION

Problem 1

The traces are evaluted by using the rules derived in page 252-253, chapter 7 Griffiths. We have:

$$\langle|\mathcal{M}|^2\rangle = \frac{g_e^4}{4(p_1-p_3)^4}\mathrm{Tr}\{\gamma^\mu(\not{p}_1+mc)\gamma^\nu(\not{p}_3+mc)\}\times\mathrm{Tr}\{\gamma_\mu(\not{p}_2+Mc)\gamma_\nu(\not{p}_4+Mc)\} \tag{1}$$

Consider the first trace in more detail. Using that the trace of an odd number of γ-matrices is zero, we get

$$\mathrm{Tr}\{\gamma^\mu(\not{p}_1+mc)\gamma^\nu(\not{p}_3+mc)\} = \mathrm{Tr}\{\gamma^\mu\not{p}_1\gamma^\nu\not{p}_3+(mc)^2\gamma^\mu\gamma^\nu\}. \tag{2}$$

The first part of the trace has four γ-matrices and the second has two γ-matrices. The trace is then evaluated using the following rules in the Griffiths book:

$$\begin{aligned}\mathrm{Tr}\{\gamma^\mu\gamma^\nu\} &= 4g^{\mu\nu},\\ \mathrm{Tr}\{\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma\} &= 4(g^{\mu\nu}g^{\lambda\sigma}-g^{\mu\lambda}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\lambda}).\end{aligned} \tag{3}$$

The second trace in Eq. (1) is evaluated in exactly the same way, after making the substitutions $m\to M, p_1\to p_2, p_3\to p_4$. Taking finally the limit $m\to 0, M\to 0$, we arrive at:

$$\langle|\mathcal{M}|^2\rangle = \frac{8g_e^4}{(p_1-p_3)^4}[(p_1p_2)(p_3p_4)+(p_1p_4)(p_2p_3)] \tag{4}$$

in the high-energy limit. The remaining part is to evaluate the products of the 4-vectors in the CM frame. Assume that θ is the scattering angle between p_1 and p_3. Due to a net momentum of zero before and after the collision, we may write:

$$p_1=(E_1/c,\mathbf{p_1}),\ p_2=(E_2/c,-\mathbf{p_1}),\ p_3=(E_3/c,\mathbf{p_3}),\ p_4=(E_4/c,-\mathbf{p_3}). \tag{5}$$

However, since the masses are neglected and energy is conserved, we have $E_j=E$ and hence $|\mathbf{p_1}|=|\mathbf{p_3}|$. Inserting this into the expression for the amplitude above, we find:

$$\langle|\mathcal{M}|^2\rangle = \frac{8g_e^4}{(p_1-p_3)^4}(E^4/c^4)[4+(1+\cos\theta)^2]. \tag{6}$$

Moreover, we have that $(p_1-p_3)^4=(2\mathbf{p}^2-2\mathbf{p}^2\cos\theta)^2=4\mathbf{p}^4(1-\cos\theta)^2$. Since we are approximating the particles as massless, we have $E=|\boldsymbol{p}|c$. Thus, we get

$$\langle|\mathcal{M}|^2\rangle = \frac{2g_e^4}{(1-\cos\theta)^2}[4+(1+\cos\theta)^2]. \tag{7}$$

Now use the trigonometric half-angle formulas:

$$2\sin^2(\theta/2) = 1-\cos\theta,\ 2\cos^2(\theta/2) = 1+\cos\theta. \tag{8}$$

This leads to:

$$\langle|\mathcal{M}|^2\rangle = \frac{2g_e^4}{\sin^4(\theta/2)}[1+\cos^4(\theta/2)]. \tag{9}$$

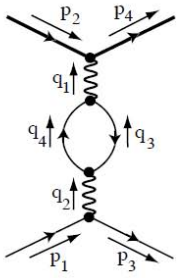
Finally, use that in the CM frame for two-body scattering (result derived previously in the lectures/book), we have:

$$\frac{d\sigma}{d\Omega} = (\hbar c/8\pi)^2 \frac{S\langle|\mathcal{M}|^2\rangle}{(E_1+E_2)^2} \tag{10}$$

Here, the statistical factor is $S=1$ and $E_1 = E_2 = E$. Inserting our expression for the amplitude into the above equation, we find:

$$\frac{d\sigma}{d\Omega} = (\hbar c/8\pi)^2 \frac{g_e^4}{2E^2} \frac{1+\cos^4\theta/2}{\sin^4\theta/2} \tag{11}$$

Problem 2



Applying the Feynman rules,

$$\int [\bar{u}(3)\,(ig_e\gamma^\mu)\,u(1)]\,\frac{-ig_{\mu\lambda}}{q_2^2}\,[\text{LOOP}]\,\frac{-ig_{\kappa\nu}}{q_1^2}\,[\bar{u}(4)\,(ig_e\gamma^\nu)\,u(2)]$$
$$\times\,(2\pi)^4\delta^4(p_1-p_3-q_2)(2\pi)^4\delta^4(q_2-q_3-q_4)(2\pi)^4\delta^4(q_3+q_4-q_1)$$
$$\times\,(2\pi)^4\delta^4(q_1+p_2-p_4)\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4q_3}{(2\pi)^4}\frac{d^4q_4}{(2\pi)^4}$$

where "LOOP" stands for

$$-\text{Tr}\left[(ig_e\gamma^\lambda)\frac{i(\not{q}_4+mc)}{q_4^2-m^2c^2}(ig_e\gamma^\kappa)\frac{i(\not{q}_3+mc)}{q_3^2-m^2c^2}\right]$$

The q_2 integral, using $\delta^4(p_1-p_3-q_2)$, sends $q_2 \to p_1-p_3 \equiv q$ (for short). The q_1 integral, using $\delta^4(q_1+p_2-p_4)$, sends $q_1 \to p_4-p_2$, and the two remaining delta functions $\Rightarrow$ $q_3+q_4 = p_1-p_3 = p_4-p_2$, so q_1 is also q (of course). The q_3 integral, using $\delta^4(q-q_3-q_4)$, sends $q_3 \to q-q_4$, and we erase the final delta function $(2\pi)^2\delta^4(p_1-p_3+p_2-p_4)$. The is still an integral over q_4, which (for simplicity) we rename k. Multiplying by i: $\mathcal{M} =$

$$-\frac{ig_e^4}{q^4}\,[\bar{u}(3)\gamma^\mu u(1)]\left\{\frac{d^4k}{(2\pi)^4}\frac{\text{Tr}\,[\gamma_\mu(\not{k}+mc)\gamma_\nu(\not{q}-\not{k}+mc)]}{[k^2-m^2c^2][(q-k)^2-m^2c^2]}\right\}[\bar{u}(4)\gamma^\nu u(2)]\ \checkmark$$

NB! The direction of the momentum $\vec{q}_3$ above is opposite to what it should be: it must be directed along the arrow on the fermion line if we are to use the expression for the fermion propagator. As a result, it should not be slashed q minus slashed k inside the integrand, but slashed k minus slashed q in the nominator (for the denominator, the order doesn't matter due to the square).