

FY3403 Particle physics

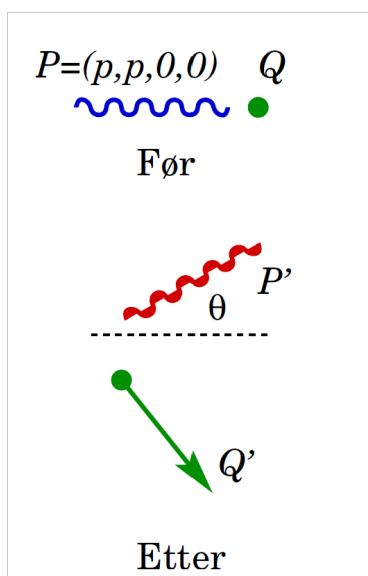
Problemset 3

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SUGGESTED SOLUTION

**Problem 1**

a) The conservation laws for energy and momentum in respectively the x - and y -direction becomes

$$p'c + \sqrt{(p^{(m)}c)^2 + (mc^2)^2} = pc + mc^2, \quad (1)$$

$$p_x^{(m)} + p'_x = p, \quad (2)$$

$$p_y^{(m)} + p'_y = 0. \quad (3)$$

b) We find $p_y^{(m)} = -p'_y$ and $p_x^{(m)} = p - p'_x$, so that

$$(p^{(m)})^2 = p_y'^2 + p_x'^2 + p^2 - 2pp'_x = p^2 + p'^2 - 2pp' \cos \theta. \quad (4)$$

The equation for energy conservation may thus be written (after division by c)

$$\sqrt{p^2 + p'^2 - 2pp' \cos \theta + (mc)^2} = p - p' + mc. \quad (5)$$

Squaring this gives

$$p^2 + p'^2 - 2pp' \cos \theta + (mc)^2 = p^2 + p'^2 + (mc)^2 + 2mcp - 2(p - mc)p'. \quad (6)$$

Solving now with respect to p' gives

$$p' = \frac{p}{1 + (p/mc)(1 - \cos\theta)}. \quad (7)$$

c) Inserting $p = h/\lambda$, $p' = h/\lambda'$ gives equation (1) in the problem text after a simple rewriting.

Note that a perhaps more elegant way to find p' is by using four-vectors. Let P, Q, P', Q' denote the four-momenta of respectively the photon and particle before and after the collision:

$$P = (p, p, 0, 0), Q = (mc, 0, 0, 0), P' = (p', p' \cos\theta, p' \sin\theta, 0). \quad (8)$$

Q' is not listed as it turns out to not be needed. To see this, use conservation of four-momentum $P + Q = P' + Q'$. Squaring this equation gives $PQ = P'Q'$ since $P^2 = P'^2 = 0$ and $Q^2 = Q'^2 = (mc)^2$. Then eliminate Q' in favor of the other four-momenta to get

$$PQ = P'(P + Q). \quad (9)$$

Explicit evaluation of the scalar products now gives the same result as we found above.

Problem 2

a) Due to time dilation, the particles lifetime (measured by a clock at rest) is prolonged by a factor $\gamma = \sqrt{1 - (v/c)^2}^{-1}$. At the same time their total energy equals $E = mc^2\gamma$ and the kinetic energy is $T = E - mc^2 = mc^2(\gamma - 1)$. For $T = 20$ MeV, we thus get:

$$\gamma = 1 + \frac{20}{105.66} = 1.189. \quad (10)$$

and

$$v = \sqrt{1 - \frac{1}{\gamma^2}}c = 0.541c. \quad (11)$$

This means that the muon needs a time $t = 8/(300000 \times 0.541) \text{ s} = 49.3 \mu\text{s}$ to reach the surface of the Earth. At the same time, their lifetime is $\tau = 1.189 \times 2.197 \mu\text{s} = 2.612 \mu\text{s}$. The probability for the muon to reach the Earth before disintegrating is thus

$$p = e^{-t/\tau} = e^{-18.855} = 6.48 \times 10^{-9}. \quad (12)$$

b) For $T = 20$ GeV, the corresponding numbers are

$$\gamma = 190.29, v = 0.99999c, t = 26.7 \mu\text{s}, \tau = 418.1 \mu\text{s} \quad (13)$$

so that the probability to reach the Earth's surface becomes

$$p = 0.938. \quad (14)$$

c) The calculation proceeds exactly as before, but we must take into account that the mass and lifetime is different. For $T = 20$ MeV, we get

$$\gamma = 1.143, v = 0.485c, t = 55013 \text{ ns}, \tau = 29.8 \text{ ns}, p = 10^{-803}. \quad (15)$$

For $T = 20$ GeV, we get

$$\gamma = 144.29, \nu = 0.99998c, t = 26667 \text{ ns}, \tau = 3756 \text{ ns}, p = 8 \times 10^{-4}. \quad (16)$$

Problem 3

a) Due to conservation of momentum the two photons must emerge in opposite directions with momenta which are equal in magnitude. Their energies are thus equal:

$$E_1 = E_2 = m_\pi c^2 / 2 = 67.488 \text{ MeV}. \quad (17)$$

b) We compute with four-vectors and use units where $c = 1$. Let p, k_1, k_2 be the four-momentum of respectively the pion and the two photons. We have, since $k_1^2 = k_2^2 = 0$,

$$p = k_1 + k_2, m_\pi^2 = p^2 = 2k_1 k_2, pk_1 = pk_2 = k_1 k_2 = m_\pi^2 / 2. \quad (18)$$

A direct computation of the scalar product gives $pk_1 = E_\pi E_1 - p_\pi E_1 \cos \theta_1$. In effect,

$$E_1 = \frac{m_\pi^2}{2(E_\pi - p_\pi \cos \theta_1)} = \frac{m_\pi^2}{2(\sqrt{p_\pi^2 + m_\pi^2} - p_\pi \cos \theta_1)} = 77.21 \text{ MeV}. \quad (19)$$

We further find that

$$E_2 = E_\pi - E_1 = \sqrt{p_\pi^2 + m_\pi^2} - E_1 = 90.79 \text{ MeV} \quad (20)$$

and

$$\cos \theta_2 = \frac{E_2 E_\pi - m_\pi^2 / 2}{E_2 p_\pi} = 0.6763. \quad (21)$$

in effect that $\theta_2 = 47.4^\circ$.

c) We use the relation

$$m_\pi^2 = (p_1 + p_2)^2 = 2p_1 p_2 = 2E_1 E_2 (1 - \cos \theta_{12}) = 4(E_\pi - E_1) E_1 \sin^2 \theta_{12} / 2. \quad (22)$$

This gives

$$\sin^2 \theta_{12} / 2 = \frac{m_\pi^2}{4E_1(E_\pi - E_1)}. \quad (23)$$

The right hand side (hence also the left hand side, and the angle θ_{12}) is smallest when $E_1 = E_2 = E_\pi / 2$. This means that

$$m_{\pi^0} = E_\pi \sin \frac{\theta_{\min}}{2}. \quad (24)$$