

FY3403 Particle physics
Problemset 10

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SUGGESTED SOLUTION

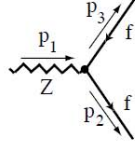
(a) Extra info about solution on next page: to perform the spin summation over outgoing spins (4th equation to 5th equation on the next page), use the result we proved in the lectures that

$$\sum_{\text{spins}} [\bar{u}(a)\Gamma_1 u(b)][\bar{u}(a)\Gamma_2 u(b)]^* = \text{Tr}\{\Gamma_1(\not{p}_b + m_b c)\bar{\Gamma}_2(\not{p}_a + m_a c)\} \quad (1)$$

and then observe that the left-hand side of the above equation is identical to the 4th equation below by using that

$$[\bar{u}(a)\Gamma_2 u(b)]^* = [\bar{u}(a)\Gamma_2 u(b)]^\dagger = [\bar{u}(b)\bar{\Gamma}_2 u(a)]. \quad (2)$$

Finally, use that if $\bar{u}(b), u(b)$ are replaced by $\bar{v}(b), v(b)$ (which is the case in the equations below), the completeness relation for the v -spinors is such that the mass $m_b \rightarrow -m_b$ on the right hand side of Eq. (1). This completes the transition from the 4th to 5th equation.



Dropping the superscript f on c_V^f and c_A^f , for the moment,

$$\mathcal{M} = i\epsilon_\mu(1)\bar{u}(2) \left[-\frac{ig_Z}{2} \gamma^\mu (c_V - c_A \gamma^5) \right] v(3) \left[(2\pi)^4 \delta^4(p_1 - p_2 - p_3) \right];$$

$$|\mathcal{M}|^2 = \left(\frac{g_Z}{2} \right)^2 \epsilon_\mu(1)\epsilon_\nu^*(1) \left[\bar{u}(2) \gamma^\mu (c_V - c_A \gamma^5) v(3) \right] \underbrace{\left[\bar{u}(2) \gamma^\nu (c_V - c_A \gamma^5) v(3) \right]^*}_{v(3) \gamma^0 [\gamma^\nu (c_V - c_A \gamma^5)]^\dagger \gamma^0 u(2)}$$

But

$$\begin{aligned} \gamma^0 [\gamma^\nu (c_V - c_A \gamma^5)]^\dagger \gamma^0 &= \gamma^0 (c_V - c_A \gamma^5)^\dagger \gamma^{\nu\dagger} \gamma^0 = \gamma^0 (c_V^* - c_A^* \gamma^5) \gamma^{\nu\dagger} \gamma^0 \\ &= (c_V^* + c_A^* \gamma^5) \gamma^0 \gamma^{\nu\dagger} \gamma^0 = (c_V^* + c_A^* \gamma^5) \gamma^\nu = \gamma^\nu (c_V^* - c_A^* \gamma^5), \end{aligned}$$

so

$$|\mathcal{M}|^2 = \left(\frac{g_Z}{2} \right)^2 \epsilon_\mu(1)\epsilon_\nu^*(1) \left[\bar{u}(2) \gamma^\mu (c_V - c_A \gamma^5) v(3) \right] \left[\bar{u}(2) \gamma^\nu (c_V^* - c_A^* \gamma^5) v(3) \right].$$

Summing over the outgoing spins, we get

$$\left(\frac{g_Z}{2} \right)^2 \epsilon_\mu(1)\epsilon_\nu^*(1) \text{Tr} \left[\gamma^\mu (c_V - c_A \gamma^5) (\not{p}_3 - m_3) \gamma^\nu (c_V^* - c_A^* \gamma^5) (\not{p}_2 + m_2) \right].$$

The trace was calculated in Problem 9.2; quoting Eq. 9.159, with $m_2 = m_3 \rightarrow 0$,

$$\begin{aligned} \left(\frac{g_Z}{2} \right)^2 \epsilon_\mu(1)\epsilon_\nu^*(1) 4 \left\{ (|c_V|^2 + |c_A|^2) \left[p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - (p_2 \cdot p_3) g^{\mu\nu} \right] \right. \\ \left. + i(c_V c_A^* - c_V^* c_A) \epsilon^{\mu\nu\lambda\sigma} p_{2\lambda} p_{3\sigma} \right\}. \end{aligned}$$

The next step is to average over the (three) spin states of the Z, using the completeness relation in Problem 9.1 (Eq. 9.158):

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{3} g_Z^2 \left[-g_{\mu\nu} + \frac{p_{1\mu} p_{1\nu}}{(M_Z c)^2} \right] (|c_V|^2 + |c_A|^2) \left[p_2^\mu p_3^\nu + p_2^\nu p_3^\mu - g^{\mu\nu} (p_2 \cdot p_3) \right].$$

(The $\epsilon^{\mu\nu\lambda\sigma}$ term gives zero, since it is antisymmetric in $\mu \leftrightarrow \nu$, whereas $g_{\mu\nu}$ and $p_{1\mu}p_{1\nu}$ are symmetric.)

$$\begin{aligned} \langle |\mathcal{M}|^2 \rangle &= \frac{1}{3} g_Z^2 (|c_V|^2 + |c_A|^2) \left\{ -2(p_2 \cdot p_3) + 4(p_2 \cdot p_3) \right. \\ &\quad \left. + \frac{1}{(M_{Zc})^2} \left[2(p_1 \cdot p_2)(p_1 \cdot p_3) - \underbrace{(p_1 \cdot p_1)(p_2 \cdot p_3)}_{(M_{Zc})^2} \right] \right\} \\ &= \frac{1}{3} g_Z^2 (|c_V|^2 + |c_A|^2) \left[(p_2 \cdot p_3) + 2 \frac{(p_1 \cdot p_2)(p_1 \cdot p_3)}{(M_{Zc})^2} \right]. \end{aligned}$$

The Golden Rule for 2-body decays in the CM frame (Eq. 6.35) says

$$\begin{aligned} \Gamma &= \frac{|\mathbf{p}|}{8\pi\hbar M_{Zc}^2} \langle |\mathcal{M}|^2 \rangle \\ &= \frac{|\mathbf{p}| g_Z^2}{24\pi\hbar M_{Zc}^2} (|c_V|^2 + |c_A|^2) \underbrace{\left[(p_2 \cdot p_3) + 2 \frac{(p_1 \cdot p_2)(p_1 \cdot p_3)}{(M_{Zc})^2} \right]}_{\diamond}. \end{aligned}$$

In the rest system of the Z ,

$$p_1 = (M_{Zc}, \mathbf{0}); \quad p_2 = \left(\frac{M_{Zc}}{2}, \mathbf{p} \right); \quad p_3 = \left(\frac{M_{Zc}}{2}, -\mathbf{p} \right),$$

and (since f is "massless")

$$\left(\frac{M_{Zc}}{2} \right)^2 - \mathbf{p}^2 = 0; \quad \text{or} \quad \mathbf{p}^2 = \left(\frac{M_{Zc}}{2} \right)^2.$$

So

$$(p_2 \cdot p_3) = \left(\frac{M_{Zc}}{2} \right)^2 + \mathbf{p}^2 = \frac{(M_{Zc})^2}{2}; \quad (p_1 \cdot p_2) = (p_1 \cdot p_3) = \frac{(M_{Zc})^2}{2},$$

and hence

$$\diamond = \frac{(M_{Zc})^2}{2} + 2 \frac{\frac{(M_{Zc})^2}{2} \frac{(M_{Zc})^2}{2}}{(M_{Zc})^2} = (M_{Zc})^2.$$

Therefore, (restoring now the superscripts on c_V and c_A)

$$\Gamma = \left[\frac{g_Z^2 (M_{Zc}^2)}{48\pi\hbar} (|c_V^f|^2 + |c_A^f|^2) \right].$$

(b) Let $A \equiv (|c_V^f|^2 + |c_A^f|^2)$. According to Table 9.1:

$$\begin{array}{lll} \nu_e, \nu_\mu, \nu_\tau : & c_V = 0.5; & c_A = 0.5; \quad A = 0.25 + 0.25 = 0.5000 \\ e, \mu, \tau : & c_V = -0.0372; & c_A = -0.5; \quad A = 0.0014 + 0.25 = 0.2514 \\ u, c, t : & c_V = 0.1915; & c_A = 0.5; \quad A = 0.0367 + 0.25 = 0.2867 \\ d, s, b : & c_V = -0.3457; & c_A = -0.5; \quad A = 0.1195 + 0.25 = 0.3695. \end{array}$$

Now

$$\Gamma_{\text{total}} = \frac{g_z^2 M_Z c^2}{48\pi\hbar} \sum_f (|c_V^f|^2 + |c_A^f|^2),$$

where the sum is over the three neutrinos, three charged leptons, u and c (but *not* the t , since it is too heavy: $m_t > M_Z/2$), d , s , and b (and in the case of the quarks, three colors):

$$\sum_f (|c_V^f|^2 + |c_A^f|^2) = 3(0.5000) + 3(0.2514) + 6(0.2867) + 9(0.3695) = 7.2999.$$

The branching ratios ($\Gamma_i/\Gamma_{\text{total}}$) are:

e, μ, τ	:	$0.2514/7.2999 = 0.0344$,	or	$\boxed{3\%}$
ν_e, ν_μ, ν_τ	:	$0.5000/7.2999 = 0.0685$,	or	$\boxed{7\%}$
u, c	:	$3(0.2867)/7.2999 = 0.1178$,	or	$\boxed{12\%}$
d, s, b	:	$3(0.3695)/7.2999 = 0.1519$,	or	$\boxed{15\%}$

Problem 10.11

Equations 10.56 and 10.57 $\Rightarrow$

$$\left(\partial_\mu + i \frac{q}{\hbar c} \boldsymbol{\tau} \cdot \mathbf{A}'_\mu \right) \psi' = S \left(\partial_\mu + i \frac{q}{\hbar c} \boldsymbol{\tau} \cdot \mathbf{A}_\mu \right) \psi. \quad [1]$$

Now, $\psi' = S\psi$, so $\partial_\mu \psi' = (\partial_\mu S)\psi + S(\partial_\mu \psi)$, and Eq. [1] becomes

$$(\partial_\mu S)\psi + \cancel{S(\partial_\mu \psi)} + i \frac{q}{\hbar c} (\boldsymbol{\tau} \cdot \mathbf{A}'_\mu) S\psi = \cancel{S(\partial_\mu \psi)} + i \frac{q}{\hbar c} S (\boldsymbol{\tau} \cdot \mathbf{A}_\mu) \psi,$$

or

$$(\partial_\mu S) + i \frac{q}{\hbar c} (\boldsymbol{\tau} \cdot \mathbf{A}'_\mu) S = i \frac{q}{\hbar c} S (\boldsymbol{\tau} \cdot \mathbf{A}_\mu).$$

Multiply by S^{-1} on the right:

$$(\partial_\mu S)S^{-1} + i \frac{q}{\hbar c} (\boldsymbol{\tau} \cdot \mathbf{A}'_\mu) = i \frac{q}{\hbar c} S (\boldsymbol{\tau} \cdot \mathbf{A}_\mu) S^{-1},$$

or

$$\boldsymbol{\tau} \cdot \mathbf{A}'_\mu = S(\boldsymbol{\tau} \cdot \mathbf{A}_\mu)S^{-1} + i \frac{\hbar c}{q} (\partial_\mu S) S^{-1}. \quad \checkmark$$