FY3403 Particle physics
Problemset 7

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SUGGESTED SOLUTION

Problem 1
The traces are evaluted by using the rules derived in page 238-240, chapter 7 Griffiths. Using *e.g.* that the trace of an odd number of γ-matrices is zero, we arrive at:

$$\langle|\mathcal{M}|^2\rangle = \frac{8g_e^4}{(p_1-p_3)^4}[(p_1p_2)(p_3p_4)+(p_1p_4)(p_2p_3)] \tag{1}$$

in the high-energy limit where we set $(m,M)\to 0$. The remaining part is to evaluate the products of the 4-vectors in the CM frame. Assume that θ is the scattering angle between p_1 and p_3. Due to a net momentum of zero before and after the collision, we may write:

$$p_1 = (E_1/c,\mathbf{p_1}),\ p_2 = (E_2/c,-\mathbf{p_1}),\ p_3 = (E_3/c,\mathbf{p_3}),\ p_4 = (E_4/c,-\mathbf{p_3}). \tag{2}$$

However, since the masses are neglected and energy is conserved, we have $E_j = E$ and hence $|\mathbf{p_1}| = |\mathbf{p_3}|$. Inserting this into the expression for the amplitude above, we find:

$$\langle|\mathcal{M}|^2\rangle = \frac{8g_e^4}{(p_1-p_3)^4}(E^4/c^4)[4+(1+\cos\theta)^2]. \tag{3}$$

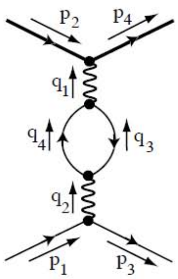
Moreover, we have that $(p_1-p_3)^4 = (2\mathbf{p}^2-2\mathbf{p}^2\cos\theta)^2 = 4\mathbf{p}^4(1-\cos\theta)^2$. Now use that in the CM frame for two-body scattering (result derived previously in the lectures/book), we have:

$$\frac{d\sigma}{d\Omega} = (\hbar c/8\pi)^2\frac{S\langle|\mathcal{M}|^2\rangle}{(E_1+E_2)^2} \tag{4}$$

Here, the statistical factor is $S=1$. Inserting our expression for the amplitude into the above equation, we find after some trigonometric manipulations that:

$$\frac{d\sigma}{d\Omega} = (\hbar c/8\pi)^2\frac{g_e^4}{2E^2}\frac{1+\cos^4\theta/2}{\sin^4\theta/2} \tag{5}$$

Problem 2



Applying the Feynman rules,

$$\int [\bar{u}(3)\,(ig_e\gamma^\mu)\,u(1)]\,\frac{-ig_{\mu\lambda}}{q_2^2}\,[\text{LOOP}]\,\frac{-ig_{\kappa\nu}}{q_1^2}\,[\bar{u}(4)\,(ig_e\gamma^\nu)\,u(2)]$$
$$\times(2\pi)^4\delta^4(p_1-p_3-q_2)(2\pi)^4\delta^4(q_2-q_3-q_4)(2\pi)^4\delta^4(q_3+q_4-q_1)$$
$$\times(2\pi)^4\delta^4(q_1+p_2-p_4)\frac{d^4q_1}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}\frac{d^4q_3}{(2\pi)^4}\frac{d^4q_4}{(2\pi)^4}$$

where "LOOP" stands for

$$-\text{Tr}\left[(ig_e\gamma^\lambda)\frac{i(\not{q}_4+mc)}{q_4^2-m^2c^2}(ig_e\gamma^\kappa)\frac{i(\not{q}_3+mc)}{q_3^2-m^2c^2}\right]$$

The q_2 integral, using $\delta^4(p_1-p_3-q_2)$, sends $q_2\to p_1-p_3\equiv q$ (for short). The q_1 integral, using $\delta^4(q_1+p_2-p_4)$, sends $q_1\to p_4-p_2$, and the two remaining delta functions $\Rightarrow$ $q_3+q_4=p_1-p_3=p_4-p_2$, so q_1 is also q (of course). The q_3 integral, using $\delta^4(q-q_3-q_4)$, sends $q_3\to q-q_4$, and we erase the final delta function $(2\pi)^2\delta^4(p_1-p_3+p_2-p_4)$. The is still an integral over q_4, which (for simplicity) we rename k. Multiplying by i: $\mathcal{M}=$

$$-\frac{ig_e^4}{q^4}[\bar{u}(3)\gamma^\mu u(1)]\left\{\frac{d^4k}{(2\pi)^4}\frac{\text{Tr}\left[\gamma_\mu(\not{k}+mc)\gamma_\nu(\not{q}-\not{k}+mc)\right]}{[k^2-m^2c^2][(q-k)^2-m^2c^2]}\right\}[\bar{u}(4)\gamma^\nu u(2)]\ \checkmark$$

NB! The direction of the momentum $\vec{q}_3$ above is opposite to what it should be: it must be directed along the arrow on the fermion line if we are to use the expression for the fermion propagator. As a result, it should not be slashed q minus slashed k inside the integrand, but slashed k minus slashed q in the nominator (for the denominator, the order doesn't matter due to the square).