FY3403 Particle physics
Problemset 5

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SUGGESTED SOLUTION

Problem 1
The general strategy for identifying the isospin classification is to look at (i) how many particles of a given type (for instance Σ) that exist with a given strangeness content, (ii) the charge of the specific particle we are asked to identify, and (iii) if the particle in question has zero up or down quarks. Let us look at some examples.

If the particle has no up or down quarks, it has isospin $I = I_3 = 0$. Therefore, the Ω-particles have zero isospin.

If we instead consider the Σ-particles, there are three of them. This means that $I = 1$ since the number of particles in an isospin multiplet should be $2I+1$. The one with highest charge is the Σ^+, and thus it has $(I, I_3) = (1, 1)$.

If we look at the Δ-particles, there are four of them. Thus, $I = 3/2$. The Δ^0 has the second lowest charge. Therefore, it is the $I_3 = -1/2$ state in that multiplet.

If we consider a meson like the $\bar{K}^0$, we know that $I = 1/2$ since the $\bar{K}^0$ and $\bar{K}^-$ belong to the same isospin multiplet, as seen in the Meson octet. The K^0 and K^+ belong to a separate isospin multiplet since their strangeness is different than $\bar{K}^0$ and $\bar{K}^-$ and strangeness is conserved in strong interactions. Since $\bar{K}^0$ has the highest charge in its multiplet, it is the $I_3 = 1/2$ state.

For the η-particle, it is an isospin singlet since η' has different strangeness content, so $I = I_3 = 0$ for η.

Following these rules, we arrive at:

(i) Ω^-: $|II_3\rangle = |00\rangle$.
(ii) Σ^+: $|II_3\rangle = |11\rangle$.
(iii) Ξ^0: $|II_3\rangle = |\frac{1}{2}\frac{1}{2}\rangle$.
(iv) Δ^0: $|II_3\rangle = |\frac{3}{2} - \frac{1}{2}\rangle$.
(v) ρ^+: $|II_3\rangle = |11\rangle$.
(vi) η: $|II_3\rangle = |00\rangle$.
(vii) $\bar{K}^0$: $|II_3\rangle = |\frac{1}{2}\frac{1}{2}\rangle$.

Problem 2
The decomposition of the states is done by looking up the Clebsch-Gordon coefficients, which can be found in the Particle Data Group webpage: `https://pdg.lbl.gov/2002/clebrpp.pdf`.

Below, I give some details for the first few problems.

a) 1. $|11\rangle|\frac{1}{2}\frac{1}{2}\rangle = |\frac{3}{2}\frac{3}{2}\rangle$.
To see this, first find the table in the above link for the $1 \times 1/2$ state (second table from the top, to the left). Then look at the $m_1 = 1, m_2 = 1/2$ row, since those are the quantum numbers we have in our state $|11\rangle|\frac{1}{2}\frac{1}{2}\rangle$. This gives a coefficient of unity for the state $j = 1, m = 1$.

2. $|10\rangle|\frac{1}{2}\frac{1}{2}\rangle = \sqrt{2/3}|\frac{3}{2}\frac{1}{2}\rangle - \sqrt{1/3}|\frac{1}{2}\frac{1}{2}\rangle$.
To see this, first find the table in the above link for the $1 \times 1/2$ state (second table from the top, to the left). Then look at the $m_1 = 0, m_2 = 1/2$ row, since those are the quantum numbers we have in our state $|10\rangle|\frac{1}{2}\frac{1}{2}\rangle$. This gives a coefficient $\sqrt{2/3}$ for the state with $j = 3/2, m = 1/2$ and a coefficient $-\sqrt{1/3}$ for the $j = 1/2, m = 1/2$ state.

3. $|1-1\rangle|\frac{1}{2}\frac{1}{2}\rangle = \sqrt{1/3}|\frac{3}{2}-\frac{1}{2}\rangle - \sqrt{2/3}|\frac{1}{2}-\frac{1}{2}\rangle$.

4. $|11\rangle|\frac{1}{2}-\frac{1}{2}\rangle = \sqrt{1/3}|\frac{3}{2}\frac{1}{2}\rangle + \sqrt{2/3}|\frac{1}{2}\frac{1}{2}\rangle$.

5. $|10\rangle|\frac{1}{2}-\frac{1}{2}\rangle = \sqrt{2/3}|\frac{3}{2}-\frac{1}{2}\rangle + \sqrt{1/3}|\frac{1}{2}-\frac{1}{2}\rangle$.

6. $|1-1\rangle|\frac{1}{2}-\frac{1}{2}\rangle = |\frac{3}{2}-\frac{3}{2}\rangle$.

7. Left hand side: $|11\rangle|\frac{1}{2}-\frac{1}{2}\rangle = \sqrt{1/3}|\frac{3}{2}\frac{1}{2}\rangle + \sqrt{2/3}|\frac{1}{2}\frac{1}{2}\rangle$.
Right hand side: $|10\rangle|\frac{1}{2}\frac{1}{2}\rangle = \sqrt{2/3}|\frac{3}{2}\frac{1}{2}\rangle - \sqrt{1/3}|\frac{1}{2}\frac{1}{2}\rangle$.

8. Left hand side: $|10\rangle|\frac{1}{2}\frac{1}{2}\rangle = \sqrt{2/3}|\frac{3}{2}\frac{1}{2}\rangle - \sqrt{1/3}|\frac{1}{2}\frac{1}{2}\rangle$.
Right hand side: $|11\rangle|\frac{1}{2}-\frac{1}{2}\rangle = \sqrt{1/3}|\frac{3}{2}\frac{1}{2}\rangle + \sqrt{2/3}|\frac{1}{2}\frac{1}{2}\rangle$.

9. Left hand side: $|10\rangle|\frac{1}{2}-\frac{1}{2}\rangle = \sqrt{2/3}|\frac{3}{2}-\frac{1}{2}\rangle + \sqrt{1/3}|\frac{1}{2}-\frac{1}{2}\rangle$.
Right hand side: $|1-1\rangle|\frac{1}{2}\frac{1}{2}\rangle = \sqrt{1/3}|\frac{3}{2}-\frac{1}{2}\rangle - \sqrt{2/3}|\frac{1}{2}-\frac{1}{2}\rangle$.

10. Left hand side: $|1-1\rangle|\frac{1}{2}\frac{1}{2}\rangle = \sqrt{1/3}|\frac{3}{2}-\frac{1}{2}\rangle - \sqrt{2/3}|\frac{1}{2}-\frac{1}{2}\rangle$.
Right hand side: $|10\rangle|\frac{1}{2}-\frac{1}{2}\rangle = \sqrt{2/3}|\frac{3}{2}-\frac{1}{2}\rangle + \sqrt{1/3}|\frac{1}{2}-\frac{1}{2}\rangle$.

b) We find from the above results, by looking at the belonging coefficients, that

$$\mathcal{M}_1 = \mathcal{M}_6 = \mathcal{M}^{(3/2)} \tag{1}$$

$$\mathcal{M}_2 = \mathcal{M}_5 = \frac{2}{3}\mathcal{M}^{(3/2)} + \frac{1}{3}\mathcal{M}^{(1/2)} \tag{2}$$

$$\mathcal{M}_3 = \mathcal{M}_4 = \frac{1}{3}\mathcal{M}^{(3/2)} + \frac{2}{3}\mathcal{M}^{(1/2)} \tag{3}$$

$$\mathcal{M}_7 = \mathcal{M}_8 = \mathcal{M}_9 + \mathcal{M}_{10} = \sqrt{2}/3(\mathcal{M}^{(3/2)} - \mathcal{M}^{(1/2)}). \tag{4}$$

c) We have essentially three unknown quantities: the absolute value of the two amplitudes $\mathcal{M}^{(3/2)}$ and $\mathcal{M}^{(1/2)}$, and the relative phase between them. We thus have the following connections between the scattering cross sections:

$$\sigma_1 = \sigma_6 = A^2 \tag{5}$$

$$\sigma_2 = \sigma_5 = (4/9)A^2 + (1/9)B^2 + (4/9)AB\cos\phi \tag{6}$$

$$\sigma_3 = \sigma_4 = (1/9)A^2 + (4/9)B^2 + (4/9)AB\cos\phi \tag{7}$$

$$\sigma_7 = \sigma_8 = \sigma_9 = \sigma_{10} = (2/9)A^2 + (2/9)B^2 - (4/9)AB\cos\phi. \tag{8}$$

This implies the following relations between the cross sections

$$\sigma_1 + \sigma_3 = 2\sigma_2 + \sigma_7. \tag{9}$$

in addition to the identities already written down.
d) When we assume $A \gg B$, we obtain the following rations between the cross-sections:

$$\sigma_3 : \sigma_7 : \sigma_2 : \sigma_1 = 1 : 2 : 4 : 9. \tag{10}$$