FY3403 Particle physics
Problemset 3

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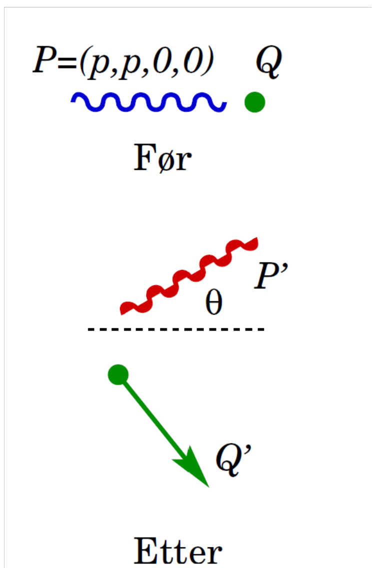
SUGGESTED SOLUTION



Problem 1

a) The conservation laws for energy and momentum in respectively the x- and y-direction becomes

$$p'c+\sqrt{(p^{(m)}c)^2+(mc^2)^2}=pc+mc^2, \tag{1}$$
$$p_x^{(m)}+p'_x=p, \tag{2}$$
$$p_y^{(m)}+p'_y=0. \tag{3}$$

b) We find $p_y^{(m)}=-p'_y$ and $p_x^{(m)}=p-p'_x$, so that

$$(p^{(m)})^2=p'^2_y+p'^2_x+p^2-2pp'_x=p^2+p'^2-2pp'\cos\theta. \tag{4}$$

The equation for energy conservation may thus be written (after division by c)

$$\sqrt{p^2+p'^2-2pp'\cos\theta+(mc)^2}=p-p'+mc. \tag{5}$$

Squaring this gives

$$p^2+p'^2-2pp'\cos\theta+(mc)^2=p^2+p'^2+(mc)^2+2mcp-2(p-mc)p'. \tag{6}$$

Solving now with respect to p' gives

$$p' = \frac{p}{1+(p/mc)(1-\cos\theta)}. \tag{7}$$

c) Inserting $p = h/\lambda, p' = h/\lambda'$ gives equation (1) in the problem text after a simple rewriting.

Note that a perhaps more elegant way to find p' is by using four-vectors. Let P, Q, P', Q' denote the four-momenta of respectively the photon and particle before and after the collision:

$$P = (p, p, 0, 0),\ Q = (mc, 0, 0, 0), P' = (p', p'\cos\theta, p'\sin\theta, 0). \tag{8}$$

Q' is not listed as it turns out to not be needed. To see this, use conservation of four-momentum $P+Q = P'+Q'$. Squaring this equation gives $PQ = P'Q'$ since $P^2 = P'^2 = 0$ and $Q^2 = Q'^2 = (mc)^2$. Then eliminate Q' in favor of the other four-momenta to get

$$PQ = P'(P+Q). \tag{9}$$

Explicit evaluation of the scalar products now gives the same result as we found above.

Problem 2

a) Due to time dilation, the particles lifetime (measured by a clock at rest) is prolonged by a factor $\gamma = \sqrt{1-(v/c)^2}^{-1}$. At the same time their total energy equals $E = mc^2\gamma$ and the kinetic energy is $T = E - mc^2 = mc^2(\gamma - 1)$. For $T = 20$ MeV, we thus get:

$$\gamma = 1 + \frac{20}{105.66} = 1.189. \tag{10}$$

and

$$v = \sqrt{1-\frac{1}{\gamma^2}}c = 0.541c. \tag{11}$$

This means that the muon needs a time $t = 8/(300000 \times 0.541)$ s = 49.3 μs to reach the surface of the Earth. At the same time, their lifetime is $\tau = 1.189 \times 2.197$ μs = 2.612 μs. The probability for the muon to reach the Earth before disintegrating is thus

$$p = e{-}t/\tau = e{-}18.855 = 6.48 \times 10^{-9}. \tag{12}$$

b) For $T = 20$ GeV, the corresponding numbers are

$$\gamma = 190.29,\ v = 0.99999c,\ t = 26.7\ \mu\text{s},\ \tau = 418.1\ \mu\text{s} \tag{13}$$

so that the probability to reach the Earth's surface becomes

$$p = 0.938. \tag{14}$$

c) The calculation proceeds exactly as before, but we must take into account that the mass and lifetime is different. For $T = 20$ MeV, we get

$$\gamma = 1.143,\ v = 0.485c,\ t = 55013\ \text{ns},\ \tau = 29.8\ \text{ns},\ p = 10^{-803}. \tag{15}$$

For $T = 20$ GeV, we get

$$\gamma = 144.29,\ v = 0.99998c,\ t = 26667\text{ ns},\ \tau = 3756\text{ ns},\ p = 8\times 10^{-4}. \tag{16}$$

Problem 3

a) Due to conservation of momentum the two photons must emerge in opposite directions with momenta which are equal in magnitude. Their energies are thus equal:

$$E_1 = E_2 = m_\pi c^2/2 = 67.488\text{ MeV}. \tag{17}$$

b) We compute with four-vectors and use units where $c = 1$. Let p, k_1, k_2 be the four-momentum of respectively the pion and the two photons. We have, since $k_1^2 = k_2^2 = 0$,

$$p = k_1 + k_2,\ m_\pi^2 = p^2 = 2k_1k_2,\ pk_1 = pk_2 = k_1k_2 = m_\pi^2/2. \tag{18}$$

A direct computation of the scalar product gives $pk_1 = E_\pi E_1 - p_\pi E_1 \cos\theta_1$. In effect,

$$E_1 = \frac{m_\pi^2}{2(E_\pi - p_\pi\cos\theta_1)} = \frac{m_\pi^2}{2(\sqrt{p_\pi^2 + m_\pi^2} - p_\pi\cos\theta_1)} = 77.21\text{ MeV}. \tag{19}$$

We further find that

$$E_2 = E_\pi - E_1 = \sqrt{p_\pi^2 + m_\pi^2} - E_1 = 90.79\text{ MeV} \tag{20}$$

and

$$\cos\theta_2 = \frac{E_2E_\pi - m_\pi^2/2}{E_2 p_\pi} = 0.6763. \tag{21}$$

in effect that $\theta_2 = 47.4^\circ$.

c) We use the relation

$$m_\pi^2 = (p_1 + p_2)^2 = 2p_1p_2 = 2E_1E_2(1 - \cos\theta_{12}) = 4(E_\pi - E_1)E_1\sin^2\theta_{12}/2. \tag{22}$$

This gives

$$\sin^2\theta_{12}/2 = \frac{m_\pi^2}{4E_1(E_\pi - E_1)}. \tag{23}$$

The right hand side (hence also the left hand side, and the angle θ_{12}) is smallest when $E_1 = E_2 = E_\pi/2$. This means that

$$m_{\pi^0} = E_\pi \sin\frac{\theta_{\min}}{2}. \tag{24}$$