FY3403 Particle physics
Problemset 2

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SUGGESTED SOLUTION

Problem 1
a) We have

$$|\psi_{\boldsymbol{p}}(0)\rangle = \cos\theta|\bar{\nu}_1\rangle + \sin\theta|\bar{\nu}_2\rangle \tag{1}$$

where $|\bar{\nu}_i\rangle$ for $i = 1,2$ represents an antineutrino with mass m_i, momentum $\boldsymbol{p}$, and energy $E_i = \sqrt{m_i^2c^4 + \boldsymbol{p}^2c^2}$. Define $\omega_i = E_i/\hbar$, then the state at time t is:

$$|\psi_{\boldsymbol{p}}(t)\rangle = \cos\theta e^{-i\omega_1 t}|\bar{\nu}_1\rangle + \sin\theta e^{-i\omega_2 t}|\bar{\nu}_2\rangle. \tag{2}$$

(b) To beautify formulae we use units where $\hbar = c = 1$ (these are called natural units). Since $|\bar{\nu}_1\rangle$ and $|\bar{\nu}_2\rangle$ are energy eigenstates we first find that

$$|\psi_{\boldsymbol{p}}(t)\rangle = \cos\theta e^{-i\omega_1 t}|\bar{\nu}_1\rangle + e^{-i\omega_2 t}\sin\theta|\bar{\nu}_2\rangle \tag{3}$$

where $\omega_i = \sqrt{\boldsymbol{p}^2 + m_i^2} \simeq |\boldsymbol{p}| + m_i^2/2\boldsymbol{p}$. Next, we have to express this state in terms of $|\bar{\nu}_e\rangle$ and $|\bar{\nu}_\mu$, which is esaily done by inverting the relation between mass and flavor eigenstates given in the problem text:

$$\begin{aligned}|\psi_{\boldsymbol{p}}(t)\rangle &= e^{-i\omega_1 t}[\cos\theta(\cos\theta|\bar{\nu}_e\rangle - \sin\theta|\bar{\nu}_\mu\rangle) + e^{-i\omega_{21}t}\sin\theta(\sin\theta|\bar{\nu}_e\rangle + \cos\theta|\bar{\nu}_\mu\rangle)] \\ &= e^{-i\omega_1 t}[(\cos^2\theta + \sin^2\theta e^{-i\omega_{21}t})|\bar{\nu}_e\rangle - \cos\theta\sin\theta(1 - e^{-i\omega_{21}t})|\bar{\nu}_\mu\rangle],\end{aligned} \tag{4}$$

where $\omega_{21} \equiv \omega_2 - \omega_1 \simeq (m_2^2 - m_1^2)/2|\boldsymbol{p}|$. In effect, we have

$$\begin{aligned}c_{\bar{e}}(t) &= e^{-i\omega_1 t}(\cos^2\theta + \sin^2\theta e^{-i\omega_{21}t}), \\ c_{\bar{\mu}}(t) &= e^{-i\omega_1 t}\cos\theta\sin\theta(e^{-i\omega_{21}t} - 1).\end{aligned} \tag{5}$$

c) We find

$$p_{\bar{e}}(t) = |c_{\bar{e}}(t)|^2 = 1 - \sin^2 2\theta\sin^2(\omega_{21}t/2) = 1 - \sin^2 2\theta\sin^2(\pi L/L_0). \tag{6}$$

We wrote $t = L/c$ and introduced the oscillation length

$$L_0 = \frac{2\pi c}{\omega_{21}} = \frac{4\pi\hbar c E_\nu}{\Delta m_{21}^2 c^4}. \tag{7}$$

For the last expression, we used that $|\boldsymbol{p}|c = E_\nu$ to a very good approximation for all detectable neutrinos. Here, $\Delta m_{21}^2 = m_2^2 - m_1^2$. Inserting numerical values we find

$$L_0[\text{km}] = 2479.684 \times \frac{E_\nu[\text{MeV}]}{\Delta m_{21}^2[(\text{meV}/c^2)^2]}. \tag{8}$$

In the real world, there are (at least) three generations of neutrinos, which makes a realistic analysis quite a lot more complicated. However, if one should use this model to predict physical effects, a reasonable parameter choice is $\Delta m_{21}^2 \simeq 100\ (\mathrm{meV}/c^2)^2$ and $\theta \simeq 34^\circ$.

Problem 2

a) We have three generations (or families) of fermions, where each generation consists of two leptons and 2×3 quarks (the factor of 3 is due to the three types of colour charge):

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}_{\mathrm{RGB}}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}_{\mathrm{RGB}}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}_{\mathrm{RGB}} \tag{9}$$

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b) (i) Baryons consist of three quarks.
(ii) Mesons consist of a quark and an antiquark.
(iii) Hadrons is a common name for strongly interacting particles (baryons and mesons).
(iv) Leptons are (as far as we know from experiments) not composed of more fundamental units.

c) (i) π^+ is a meson ($u\bar{d}$).
(ii) Δ^- is a baryon (ddd).
(iii) n is a baryon (udd).
(iv) τ is a lepton.
(v) $\bar{\nu}_\mu$ is an (anti-)lepton.
(vi) K^+ is a meson ($u\bar{s}$).
(vii) Ω^- is a baryon (sss).
(viii) Ξ^+ is nothing – only the baryons Ξ^- (ssd) and Ξ^0 (ssu) exist.

d) (i) The gluons g carry the strong interaction.
(ii) The photon γ carry the electromagnetic interaction.
(iii) $W^\pm$ carry weak charged interaction.
(iv) Z^0 carry weak neutral interaction.

In addition one believes that there exist gravitons, which carry the gravitational interaction. But gravitation is not part of the Standard Model.

Problem 3

1. Absolutely impossible! Violates conservation of electric charge.
2. Possible if the electron-positron pair has sufficient (collision) energy; proceeds through electromagnetic (γ) and neutral weak (Z^0) interaction.
3. Impossible (in vacuum); violates conservation of energy-momentum.
4. Possible; proceeds through a combination of weak charged (decay of the s-quark) and strong (to create the pion) interactions.
5. Absolutely impossible! Violates conservation of angular momentum (the left hand side must have half-integer spin, the right hand side integer spin). Also violates conservation of muon number.
6. Possible; proceeds through weak charged interaction.
7. Absolutely impossible. Violates conservation of angular momentum (the left hand side must have integer spin, the right hand side half-integer spin). Also violates conservation of baryon number.

8. Possible; proceeds through electromagnetic interaction.
9. Almost impossible; violates invariance under charge conjugation. In principle there should be a very-very low probability that the process could proceed through weak interactions, but in all likelyhood it has decayed to two photons long before that happen.
10. Possible; proceeds through weak charged interaction.
11. Possible; proceeds through weak charged interaction.