FY3403 Particle physics
Problemset 10

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SUGGESTED SOLUTION

Problem 8.16

(a) Here

$$c_1 = c_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad c_2 = c_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

From Eq. 8.47,

$$f = \frac{1}{4}\left[(0\ 1\ 0)\,\lambda^\alpha \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right]\left[(0\ 0\ 1)\,\lambda^\alpha \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right] = \frac{1}{4}\lambda^\alpha_{22}\lambda^\alpha_{33}.$$

The only λ matrix with a nonzero entry in the 33 position is λ^8, so

$$f = \frac{1}{4}\lambda^8_{22}\lambda^8_{33} = \frac{1}{4}\left(\frac{1}{\sqrt{3}}\right)\left(-\frac{2}{\sqrt{3}}\right) = -\frac{1}{6}. \quad \checkmark$$

(b)

$$c_1 = c_2 = \begin{pmatrix}1\\0\\0\end{pmatrix}, \text{ for the first term, } c_1 = c_2 = \begin{pmatrix}0\\1\\0\end{pmatrix} \text{ for the second :}$$

$$f = \frac{1}{4}\frac{1}{\sqrt{2}}\left\{\left[c_3^\dagger \lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\ 0\ 0)\,\lambda^\alpha c_4\right] - \left[c_3^\dagger \lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\,\lambda^\alpha c_4\right]\right\}$$

The outgoing quarks are in the same color state $(r\bar{r} - b\bar{b})/\sqrt{2})$, so

$$c_3 = c_4 = \begin{pmatrix}1\\0\\0\end{pmatrix}, \text{ for the first term, } c_3 = c_4 = \begin{pmatrix}0\\1\\0\end{pmatrix} \text{ for the second.}$$

There are 4 terms in all:

$$\begin{aligned} f = \frac{1}{4}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\Bigg\{ & \left[(1\ 0\ 0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\ 0\ 0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right] \\ & - \left[(0\ 1\ 0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right] \\ & - \left[(1\ 0\ 0)\,\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\,\lambda^\alpha \begin{pmatrix}1\\0\\0\end{pmatrix}\right] \\ & + \left[(0\ 1\ 0)\,\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\,\lambda^\alpha \begin{pmatrix}0\\1\\0\end{pmatrix}\right]\Bigg\} \end{aligned}$$

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$$
\begin{aligned}
f &= \frac{1}{8}\left(\lambda^{\alpha}_{11}\lambda^{\alpha}_{11} - \lambda^{\alpha}_{21}\lambda^{\alpha}_{12} - \lambda^{\alpha}_{12}\lambda^{\alpha}_{21} + \lambda^{\alpha}_{22}\lambda^{\alpha}_{22}\right) = \frac{1}{8}\left[\left(\lambda^{3}_{11}\lambda^{3}_{11} + \lambda^{8}_{11}\lambda^{8}_{11}\right)\right. \\
&\quad \left. - \left(\lambda^{1}_{21}\lambda^{1}_{12} + \lambda^{2}_{21}\lambda^{2}_{12}\right) - \left(\lambda^{1}_{12}\lambda^{1}_{21} + \lambda^{2}_{12}\lambda^{2}_{21}\right) + \left(\lambda^{3}_{22}\lambda^{3}_{22} + \lambda^{8}_{22}\lambda^{8}_{22}\right)\right] \\
&= \frac{1}{8}\left\{\left[(-1)(-1) + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\right] - [(1)(1) + (i)(-i)]\right. \\
&\quad \left. - [(1)(1) + (-i)(i)] + \left[(-1)(-1) + \left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\right]\right\} \\
&= \frac{1}{8}\left(1 + \frac{1}{3} - 1 - 1 - 1 - 1 + 1 + \frac{1}{3}\right) = \frac{1}{8}\left(-\frac{4}{3}\right) = -\frac{1}{6}. \quad \checkmark
\end{aligned}
$$

(c)

$$
\begin{aligned}
f &= \frac{1}{4}\frac{1}{\sqrt{6}}\left\{\left[c_3^{\dagger}\lambda^{\alpha}\begin{pmatrix}1\\0\\0\end{pmatrix}\right]\left[(1\ 0\ 0)\,\lambda^{\alpha}c_4\right] + \left[c_3^{\dagger}\lambda^{\alpha}\begin{pmatrix}0\\1\\0\end{pmatrix}\right]\left[(0\ 1\ 0)\,\lambda^{\alpha}c_4\right]\right. \\
&\qquad \left. - 2\left[c_3^{\dagger}\lambda^{\alpha}\begin{pmatrix}0\\0\\1\end{pmatrix}\right]\left[(0\ 0\ 1)\,\lambda^{\alpha}c_4\right]\right\}
\end{aligned}
$$

There are nine terms,

$$
\begin{aligned}
f &= \frac{1}{4}\frac{1}{\sqrt{6}}\frac{1}{\sqrt{6}}\left[\left(\lambda^{\alpha}_{11}\lambda^{\alpha}_{11} + \lambda^{\alpha}_{21}\lambda^{\alpha}_{12} - 2\lambda^{\alpha}_{31}\lambda^{\alpha}_{13}\right) + \left(\lambda^{\alpha}_{12}\lambda^{\alpha}_{21} + \lambda^{\alpha}_{22}\lambda^{\alpha}_{22} - 2\lambda^{\alpha}_{32}\lambda^{\alpha}_{23}\right)\right. \\
&\qquad \left. - 2\left(\lambda^{\alpha}_{13}\lambda^{\alpha}_{31} + \lambda^{\alpha}_{23}\lambda^{\alpha}_{32} - 2\lambda^{\alpha}_{33}\lambda^{\alpha}_{33}\right)\right] \\
&= \frac{1}{24}\left\{\left(1 + \tfrac{1}{3}\right) + (1+1) - 2(1+1) + (1+1) + \left(1 + \tfrac{1}{3}\right) - 2(1+1)\right. \\
&\qquad \left. - 2\left[(1+1) + (1+1) - 2\left(\tfrac{4}{3}\right)\right]\right\} \\
&= \frac{1}{24}\left(\tfrac{4}{3} + 2 - 4 + 2 + \tfrac{4}{3} - 4 - 4 - 4 + \tfrac{16}{3}\right) = \frac{1}{24}(-4) = -\frac{1}{6}. \quad \checkmark
\end{aligned}
$$

Problem 10.11

Equations 10.56 and 10.57 $\Rightarrow$

$$\left(\partial_\mu + i\frac{q}{\hbar c}\boldsymbol{\tau}\cdot\mathbf{A}'_\mu\right)\psi' = S\left(\partial_\mu + i\frac{q}{\hbar c}\boldsymbol{\tau}\cdot\mathbf{A}_\mu\right)\psi. \qquad [1]$$

Now, $\psi' = S\psi$, so $\partial_\mu\psi' = (\partial_\mu S)\psi + S(\partial_\mu\psi)$, and Eq. [1] becomes

$$(\partial_\mu S)\psi + \cancel{S(\partial_\mu\psi)} + i\frac{q}{\hbar c}\left(\boldsymbol{\tau}\cdot\mathbf{A}'_\mu\right)S\psi = \cancel{S(\partial_\mu\psi)} + i\frac{q}{\hbar c}S\left(\boldsymbol{\tau}\cdot\mathbf{A}_\mu\right)\psi,$$

or

$$(\partial_\mu S) + i\frac{q}{\hbar c}\left(\boldsymbol{\tau}\cdot\mathbf{A}'_\mu\right)S = i\frac{q}{\hbar c}S\left(\boldsymbol{\tau}\cdot\mathbf{A}_\mu\right).$$

Multiply by S^{-1} on the right:

$$(\partial_\mu S)S^{-1} + i\frac{q}{\hbar c}\left(\boldsymbol{\tau}\cdot\mathbf{A}'_\mu\right) = i\frac{q}{\hbar c}S\left(\boldsymbol{\tau}\cdot\mathbf{A}_\mu\right)S^{-1},$$

or

$$\boldsymbol{\tau}\cdot\mathbf{A}'_\mu = S(\boldsymbol{\tau}\cdot\mathbf{A}_\mu)S^{-1} + i\frac{\hbar c}{q}\left(\partial_\mu S\right)S^{-1}. \checkmark$$