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Suggested solution for autumn 2025 exam FY3403: Particle Physics

NOTE: The solutions below are meant as guidelines for how the problems may be solved and do not necessarily contain all the detailed steps of the calculations.

PROBLEM 1

(a) The total angular momentum is zero in the initial state since the kaon has zero spin and we're in the CoM frame. Pions are spinless. Hence, their total angular momentum has to be zero after the decay in order to conserve angular momentum.

(b) Addition of two isospins 1 can give isospin either 0, 1, or 2.

(c) There are $3\times 3=9$ possible states, from which we can make six symmetric states

$$|++\rangle,\ |00\rangle\ |--\rangle,\ |0+\rangle+|+0\rangle,\ |+-\rangle+|-+\rangle\ |0-\rangle+|-0\rangle, \tag{1}$$

and three antisymmetric states

$$|0+\rangle-|+0\rangle,\ |+-\rangle-|-+\rangle\ |0-\rangle-|-0\rangle. \tag{2}$$

We know that there is $2\times 0+1=1$ states with isospin 0, $2\times 1+1=3$ states with isospin 1, and $2\times 2+1=5$ states with isospin 2.

(d) Since all two-pion states in this problem with the same total isospin have the same symmetry under exchange of particles, it follows that it must be the total isospin 0 and isospin 2 states (6 in total) which are symmetric whereas the isospin 1 state (3 in total) are antisymmetric.

(e) The two-pion state must be described by a two-boson wavefunction. Such a wavefunction is symmetric under exchange of particles. If the spatial part is symmetric, the isospin part must also be symmetric. Acccording to our reasoning in (d) above, this excludes the possibility of a total isospin 1 state. Note that the pions are spinless, and hence the spin part of the wavefunction makes no difference.

PROBLEM 2

(a) The fermion must have charge +1/3 due to conservation of charge, which means that $\bar{f}$ must be an anti-quark of the type $\bar{d}, \bar{s}$, or $\bar{b}$.

(b) The amplitude and diagram are given below:

$$-\mathrm{i}\mathcal{M}=\frac{(-\mathrm{i}g_W)(\mathrm{i}g_c)}{2\sqrt{2}}[\bar{v}_1\gamma^\mu(1-\gamma_5)u_2]\frac{-\mathrm{i}(g_{\mu\nu}-q_\mu q_\nu/M_W^2)}{q^2-M_W^2}[\bar{u}_3\gamma^\nu(1-\gamma_5)v_4]. \tag{3}$$

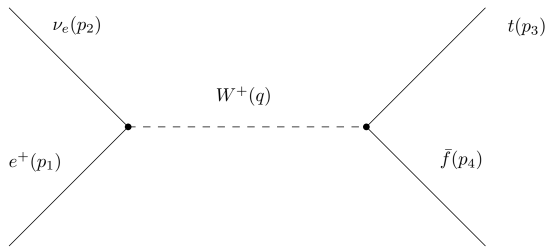

FIG. 1: Feynman diagram.

(c) Let us introduce some convenient notation known as Feynman *slash* notation: $\not{a} \equiv a^\mu \gamma_\mu$, $\not{a}^* \equiv \gamma^\mu a^*_\mu$, and $\bar{\Gamma} \equiv \gamma^0 \Gamma^\dagger \gamma^0$ for a matrix Γ. We set $c = 1$ below for simplicity. Our starting point is:

$$|\mathcal{M}|^2 = \frac{g_e^4}{(p_1 - p_3)^4} [\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu u(2)][\bar{u}(3)\gamma^\nu u(1)]^*[\bar{u}(4)\gamma_\nu u(2)]^*. \tag{4}$$

This mathematical object is built up from the schematic structure: $G \equiv [\bar{u}(a)\Gamma_1 u(b)][\bar{u}(a)\Gamma_2 u(b)]^*$, where $\Gamma_{1,2}$ are 4×4 matrices. For a scalar (1×1 matrix) the operation * is equivalent to †, which means that

$$[\bar{u}(a)\Gamma_2 u(b)]^* = [u^\dagger(a)\gamma^0 \Gamma_2 u(b)]^\dagger = u^\dagger(b)\Gamma_2^\dagger \gamma^{0\dagger} u(a) = \bar{u}(b)\bar{\Gamma}_2 u(a) \tag{5}$$

where we used that $\gamma^{0\dagger} = \gamma^0$ and $(\gamma^0)^2 = 1$. We now perform a summation over the spin orientations of particle b:

$$\sum_{b \text{ spins}} G = \bar{u}(a)\Gamma_1 \left\{ \sum_{s_b = 1,2} u^{(s_b)}(p_b)\bar{u}^{(s_b)}(p_b) \right\} \bar{\Gamma}_2 u(a) = \bar{u}(a)\Gamma_1(\not{p}_b + m_b)\bar{\Gamma}_2 u(a) = \bar{u}(a) Q u(a), \tag{6}$$

where $Q \equiv \Gamma_1(\not{p}_b + m_b c)\bar{\Gamma}_2$. Similarly, for a summation over the a spins we get:

$$\sum_{a \text{ spins}} \sum_{b \text{ spins}} \bar{u}_i^{(s_a)} Q_{ij} u_j^{(s_a)} = Q_{ij} \left\{ \sum_{s_a = 1,2} u^{(s_a)} \bar{u}^{(s_a)} \right\}_{ji} = Q_{ij}(\not{p}_a + m_a c)_{ji} = \mathrm{Tr}\{Q(\not{p}_a + m_a c)\}. \tag{7}$$

Here, $\mathrm{Tr}(A) = \sum_i A_{ii}$. We also used that u and $\bar{u}$ are 4×1 and 1×4 spinors, respectively. In total, we thus have

$$\sum_{\text{all spins}} [\bar{u}(a)\Gamma_1 u(b)][\bar{u}(a)\Gamma_1 u(b)]^* = \mathrm{Tr}\{\Gamma_1(\not{p}_b + m_b c)\bar{\Gamma}_2(\not{p}_a + m_a c)\}. \tag{8}$$

We see that there are no spinors left after the summation over spins: only matrix multiplication and a trace at the end. This procedure is known as *Casimir's trick*. Note that if u is replaced with v, the corresponding mass should change sign.

In our particular case, we have $\Gamma_2 = \gamma^\nu$ and $\bar{\Gamma}_2 = \gamma^0 \gamma^{\nu\dagger} \gamma^0 = \gamma^\nu$. We find:

$$\langle |\mathcal{M}|^2 \rangle = \frac{g_4^4}{4(p_1 - p_3)^4} \mathrm{Tr}\{\gamma^\mu(\not{p}_1 + mc)\gamma^\nu(\not{p}_3 + mc)\} \mathrm{Tr}\{\gamma_\mu(\not{p}_2 + Mc)\gamma_\nu(\not{p}_4 + Mc)\}, \tag{9}$$

where $m = m_e$, $M = m_\mu$, and $\frac{1}{4}$ is included to average over initial spins (2 particles with 2 configurations each = 4 configurations).

PROBLEM 3

(a) If we demand that the Dirac Lagrangian is invariant under a local U(1) transformation $\psi \to \psi e^{i\theta(x)}$, taking the Dirac Lagrangian on its own violates such invariance since it transforms as

$$\mathcal{L} \to \mathcal{L} - \hbar c(\partial_\mu \theta)\bar{\psi}\gamma^\mu \psi. \tag{10}$$

However, we can fix this by adding a term $\propto J^\mu A_\mu$ to the Lagrangian where $J^\mu \propto q\bar{\psi}\gamma^\mu\psi$ is the conserved electric 4-current carried by Dirac fermions. If we now demand that A_μ also transforms by acquiring a term $\propto \partial_\mu\theta(x)$, the Lagrangian is gauge-invariant. Since the massless Proca-Lagrangian is invariant under such a transformation, we can add that as the free Lagrangian of the field A_μ. Hence, we see that local gauge invariance of the Dirac equation generates a coupling to a massless spin-1 boson: the photon.

Above, $\propto$ has been used rather than inserting the explicit coefficients since it is not expected that the students should remember all prefactors, but rather the general strategy.

(b) The neutrino flavor states $\nu_{e,\mu,\tau}$ are not eigenstates of the free part of the weak interaction Hamiltonian, which describes their propagation through space. Instead, mass states $\nu_{1,2,3}$ are. This means that a ν_e produced in some reaction has a finite probability of changing into a different flavor $\nu_{\mu,\tau}$ as time advances. This is called neutrino oscillations.

(c) First one must be weak since it changes strangeness. Second one must also be weak since it involves an antineutrino. The third one is dominated by the strong interaction, even though it can also occur via *e.g.* electromagnetic interaction.

PROBLEM 4

(a) Lee and Yang studied β-decay of Co-atoms and found that when the Co-atom had all their spins aligned, the electrons in the decay emerged in a preferred direction. This proves parity violation, since spin is invariant under a parity transformation whereas the electron trajectory is not.

CP-violation was proven by measuring the decay of kaons moving along a long beam trajectory by Cronin and Fitch.

(b) Asymptotic freedom is the phenomenon that interactions between particles becomes weaker as the energy scale increases and the corresponding length scale decreases. This occurs in QCD, where quarks can move around more or less freely inside nucleons, but cannot escape to become free particles since the strong interaction becomes larger with separation distance. The net effect is color confinement.

(c) A gauge theory is a field theory in which the Lagrangian is invariant under a local transformation of the field(s) - these transformations are usually continuous. This symmetry gives rise to a degree of freedom which can be altered without changing the physics of the system. The different choices exploiting this degree of freedom are called gauges.