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Suggested solution for 2019 Exam in Electricity and Magnetism

NOTE: The solutions below are meant as guidelines for how the problems may be solved and do not necessarily contain all the detailed steps of the calculations.

1: Use $U = e^2/4\pi\varepsilon_0 r$ to find 1.44 meV.

2: Can be solved by constructing the effective capacitance of the junction. The parallell coupling between C and $2C$ gives an effective capacitance of $3C$. This effective capacitance in series with the C-capacitor gives a total effective capacitance $C_{\text{eff}} = 3C/4$. The charge on C_{eff} is $Q = V_0 C_{\text{eff}}$. Since the charge on capacitors coupled in series must be the same, this charge Q also sits on the effective capacitance $3C$. In turn, Q is split into $2Q/3$ on the $2C$ capacitor and $Q/3$ on the C capacitor. Therefore, the charge indicated in the picture is $CV_0/2$.

3: From a simple closed circuit with an AC voltage source and a capacitor, the student should be able to use that $v = q/C$ and the reactance $V/I = X$ to show that $X = 1/(\omega C)$. Inserting numbers gives the correct answer 0.01 Ω.

4: All surplus charge on a conductor must under equilibrium conditions reside on its surface. Therefore, the charge of the sphere will be zero.

5: Using a phasor diagram, the student should be able to derive that the impedance Z of the circuit is maximal at the resonance frequency $\omega = 1/(LC)$. Therefore, the current $I(t)$ is minimal at this frequency. I_L decreases with ω while I_c increases with ω.

6: All statements are correct.

7: Use Gauss law for a cylinder with radius r and height l to find $E(r)2\pi rl = \lambda l/\varepsilon_0$, so that $E(r) = 2k\lambda/r$.

8: As the magnet falls, a current is induced in the loop which tries to counteract the change in magnetic flux (which is increasing). This current thus has to circulate counterclockwise, which is in the negative current direction according to the arrows on the figure. When the magnet has passed through the loop, the flux decreases and a current is induced which tries to keep the flux stable: this has to circulate in the clockwise direction, which is in the positive current direction. The correct figure is thus (2).

9: It is the distance from the element dl to the point P, so the correct answer is $\mathbf{r}_3$.

10: Only the electric field, since all other quantities mentioned in the statements depend on d: $C = \varepsilon_0 A/d$, $U = \frac{1}{2}CV^2$, $V = Ed$.

11: Using the right-hand rule with the Lorentz-force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, one finds that positive charge accumulates at the top of the rod. The correct answer is thus figure C.

12: It propagates in the positive y-direction and the speed of light in the medium is found from the ratio E/B which gives about 57% of the vacuum speed of light.

13: Using Ampere's law, one easily sees that the field produced by a very long (infinite) wire scales with the current: $B \propto I$. Since the flux $\Phi \propto B$ and $\varepsilon = -d\Phi/dt$, it is clear that the current $I = \varepsilon/R$ in the conductor is proportional to k. Since the current tries to counteract the flux which induced it, it has to circulate clockwise.

14: As provided in the formula sheet, the voltage of a capacitor coupled in series with a current-source lags the current by $\pi/2$. Opposite for an inductor. Since $V_R \propto I$, alternative 2 is correct.

15: The speed of an EM wave in a medium is generally $1/\sqrt{\varepsilon\mu}$ and thus the statement about the speed of an EM wave in the alternatives is incorrect.

16: The magnetic field of a wire circulates the wire according to the right-hand rule: thumbs in the direction of I will give fingers that bend in the direction of $\mathbf{B}$. Moreover, the contribution to the total field at a given point is strongest from the wire which is closest to that point. Using these two facts in the left region, the region between the wires, and the right region, one sees that alternative 3 is correct.

17: Since the ideal inductor has no internal resistance and gives a voltage drop proportional to dI/dt, it has no effect for a dc current.

18: Since the velocity of the electric current is given by v_d, there is no drift velocity when the current is zero. Thus, the statement that the drift velocity is non-zero even when no current flows through the circuit is incorrect.

19: Diamagnetism, since it occurs even for a constant field (unlike Faraday's law).

20: Since $X = V/I$, the student should be able to derive that $X = \omega L$ from the provided formula $v = Ldi/dt$ in the formula sheet.

Problem 21: (a) The surface of the sphere is $A = 4\pi r^2$. The intensity for visible light is thus $0.05 \times 75\text{W}/(4\pi r^2) = 330$ W/m^2. (b) The electric field amplitude is related to the intensity of the entire EM wave via $E_{\text{max}} = \sqrt{2I/\varepsilon_0 c} = 500$ V/m. The magnetic field amplitude is then obtained as $B_{\text{max}} = E_{\text{max}}/c = 1.7\mu$T. We've used the speed of light in vacuum. **NB!** There might have been confusion about which intensity to use in (b): the intensity for the visible light or the full intensity for the EM wave. Thus, full score has been given if the student has used either of these two alternatives.

Problem 22: (a) Kirchhoff's law for voltage gives that

$$\varepsilon - iR - L\frac{di}{dt} = 0. \tag{1}$$

The current is thus

$$i = \frac{\varepsilon}{R}[1 - \mathrm{e}^{-(R/L)t}] \tag{2}$$

as obtained by solving the differential equation with the initial condition that $i = 0$. The student is expected that be able to deduce that this is the required initial condition, since a finite current at $t = 0$ would give an infinite derivative di/dt. At $t = 0$, $i = 0$, and therefore

$$di/dt|_{t=0} = \varepsilon/L = 2.4 \text{ A/s}. \tag{3}$$

(b) Generally, we have

$$di/dt = (\varepsilon - iR)/L. \tag{4}$$

Plugging in values and $i = 0.5$ A, we get $di/dt = 0.8$ A/s. (c) Using Eq. (2), we get 0.41 A. (d) At steady state, we have $t \to \infty$ and $di/dt \to 0$, so that $\varepsilon - iR = 0$. Therefore, $i(t \to \infty) = 0.75$ A.

Problem 23: The key here is to first realize that the electric field at a distance R from the center of the solenoid can be obtained using Faraday's law $\varepsilon = -d\Phi/dt$ since $\varepsilon = E \times 2\pi R$. The corresponding flux is computed by using that the magnetic field only exists inside the solenoid, so that $\Phi = B\pi r^2$ where $r = 1.1$ cm.

It remains to specify what the magnetic field inside the solenoid. The student is expected to be able to use Ampere's law and deduce that

$$B = \mu_0 ni \tag{5}$$

where $n = 400$ is the number of windings per meter and i is the current through each winding. We thus obtain

$$|\varepsilon| = |d\Phi/dt| = |d/dt(\mu_0 n i \pi r^2)| = \mu_0 n \pi r^2 |di/dt| = E 2\pi R. \tag{6}$$

We thus obtain $|di/dt| = 9.21$ A/s.

Problem 24: The torque is given by $\tau = \mu \times \mathbf{B}$. The maximum torque is obtained when the plane of the ring is oriented parallell to the magnetic field, so that $\mu \perp \mathbf{B}$. The value of the torque is then

$$\mu B = [I\pi(d/2)^2]B \tag{7}$$

where $B = 0.375$ T. We need to know I and this is obtained by using Ampere's law to deduce the the following relation between the field b created in the middle of the ring by the current itself and the current I:

$$b = \mu_0 I/2R. \tag{8}$$

Since $b = 75.4\mu$T, one finds $I = 3$ A. This finally yields the maximal torque $\tau = \mu B = 2.2 \times 10^{-3}$ Nm.

Problem 25: The charge q on each capacitor has to be the same since they are coupled in series. Moreover, the effective capacitance of the circuit is given by

$$1/C_{\text{eff}} = 1/C_1 + 1/C_2 + 1/C_3 \tag{9}$$

where C_i are the capacitances of the three capacitors. In this way, one finds $C_{\text{eff}} = 4.6$ pF. From now on, let $C \equiv C_{\text{eff}}$. The energy stored on the capacitors can now be computed from the single effective capacitance which holds a charge q, according to

$$U = q^2/2C. \tag{10}$$

Using Kirchhoff's law for voltage, one finds

$$q/C + iR = 0 \rightarrow q/C + (dq/dt)R = 0 \tag{11}$$

where q is the instantaneous charge on the effective capacitor. This equation is solved to give

$$q = Q_0 \mathrm{e}^{-t/(RC)} \tag{12}$$

where $Q_0 = 3.5$ nC is the charge the capacitor initially holds. The current in the circuit is thus

$$|i| = |dq/dt| = (Q_0/RC)\mathrm{e}^{-t/(RC)}. \tag{13}$$

Now, when the capacitor has lost 80% of its initial stored energy, the stored energy is equal to $0.2Q_0^2/2C$. At the time t when this happens, we thus have:

$$Q_0^2 \mathrm{e}^{-2t/(RC)}/2C = 0.2Q_0^2/2C, \tag{14}$$

which we can solve for t to give

$$t = 92.9 \text{ ps}. \tag{15}$$

The current at that time may then be computed according to Eq. (13) and gives

$$I = 13.6 \text{ A}. \tag{16}$$

Problem 26: (a) The key here is to use Gauss' law with a spherical surface to make use of the radial symmetry of the problem. For such a sphere with radius r, we get

$$Q_{\text{encl}} = \int_a^r \rho(r')dV = 4\pi\alpha \int_a^r r'dr' = 2\pi\alpha(r^2 - a^2). \tag{17}$$

According to Gauss' law, this should equal $E4\pi r^2$ so that

$$E = \frac{\alpha}{2\varepsilon_0}(1 - \alpha^2/r^2). \tag{18}$$

(b) The electric field provided by the point charge is $E_q = q/4\pi\varepsilon_0 r^2$. We see that the point charge cancels out the $1/r^2$ dependence of the E-field from the spherical shell if

$$q = 2\pi\alpha a^2. \tag{19}$$

Only the constant term in Eq. (18) then remains, so that

$$E = \frac{\alpha}{2\varepsilon_0}. \tag{20}$$