CLASSICAL MECHANICS TFY4345 - Solution Exercise Set 1

(1a) Choose the zero reference level for potential energy $V = 0$ at $r \to \infty$. In that case, $T + V = E = 0$ for a particle that barely escapes the gravitational pull of Earth. The graviational potential is:

$$V(r) = -\gamma \frac{Mm}{r} \tag{1}$$

where M is the mass of Earth and m is the particle mass. The escape velocity is then provided by:

$$\frac{1}{2} m v_0^2 - \gamma \frac{Mm}{r} = 0, \tag{2}$$

so that $v_0^2 = 2\gamma M/R$. Inserting numbers, we obtain $v_0 = 11.2$ km/s.

(1b) Assuming a constant mass gives us $\mathbf{F} = m\dot{\mathbf{v}}$, so that $\mathbf{F} \cdot \mathbf{v} = d(mv^2/2)/dt = dT/dt$. With a variable mass we have $\mathbf{F} = \dot{\mathbf{p}}$ and thus $\mathbf{F} \cdot \mathbf{p} = d(p^2/2)/dt = d(mT)/dt$ since $T = p^2/2m$.

(c) We have $V(x_1, x_2) = k(x - l)^2/2$. Lagrange's equations used on x_1 and x_2 give us:

$$\begin{aligned} \ddot{x}_1 &= -k(x-l)/m_1, \\ \ddot{x}_2 &= k(x-l)/m_2. \end{aligned} \tag{3}$$

Combining these equations gives $\ddot{R} = 0$ and $\ddot{x} = -k(x-l)/\mu$ where $\mu^{-1} = m_1^{-1} + m_2^{-1}$. The solution for $x - l$ then reads $x - l = A \sin \sqrt{k/mu}(t - t_0)$ where A is a constant. The energy E at the maximum amplitude of the oscillation will consist of purely potential energy (since the particle has no velocity at the turning point), so that $E = kA^2/2$. In effect, $x - l = \sqrt{2E/k} \sin \sqrt{k/\mu}(t - t_0)$. For $k \to \infty$, the angular frequency diverges so that $x \to l$. This means that the particles are frozen at a distance l from each other.