CLASSICAL MECHANICS TFY4345 - Exercise 6

(1a) A particle with mass m moves in a central field with force $f(r) = -k/r^2 + \beta/r^3$, β =constant. Here, the last β-term gives the deviation from Kepler. Show that the trajectory may be written as the form:

$$p/r = 1 + \varepsilon\cos(\gamma\theta) \tag{1}$$

where $\gamma \neq 1$ corresponds to an ellipse with precession ($\gamma = 1$ gives a stable ellipse). Find p, ε, and γ expressed in terms of the original parameters in the problem, such as mass m, angular momentum l, energy E et.c.

(1b) Given a spherical pendulum, mass m, length l. The polar angle is θ while ϕ is the azimuthal angle. Choose the zero level for potential energy at $\theta = \pi/2$. Show that Lagrange's equations yield

$$p_\phi = ml^2\sin^2\theta\dot{\phi} = \text{ constant}, \; \ddot{\theta} - \frac{1}{2}\sin 2\theta\dot{\phi}^2 + g\sin\theta/l = 0. \tag{2}$$

Also show that the total energy is

$$E = \frac{1}{2}ml^2\dot{\theta}^2 + V_{\text{eff}}(\theta), \tag{3}$$

and how the time and angle can be written on integral form as

$$t = \sqrt{ml^2/2}\int d\theta/\sqrt{E - V_{\text{eff}}(\theta)},$$

$$\phi = \frac{p_\phi}{\sqrt{2ml^2}}\int d\theta/(\sin^2\theta\sqrt{E - V_{\text{eff}}(\theta)}). \tag{4}$$

What kind of motion corresponds to $p_\phi = 0$?

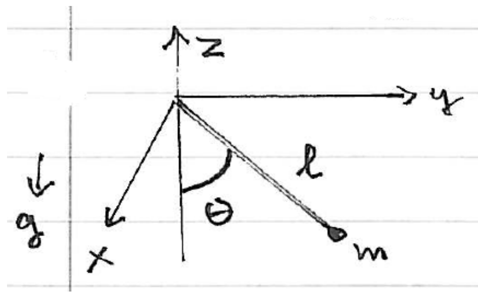

FIG. 1: (Color online). The system under consideration.