CLASSICAL MECHANICS TFY4345 - Exercise 13

(1a) The Hamiltonian for a particle with mass m is in cylindrical coordinates (r, θ, z) given by:

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} + \frac{p_z^2}{2m} + V. \quad (1)$$

Assume that the potential is separable in the following way: $V = a(r) + b(z)$, where $a(r)$ and $b(z)$ are known functions. Put $S(q, \alpha, t) = W(q, \alpha) - \alpha t$ into the Hamilton-Jacobi equation:

$$H(q, \frac{\partial S}{\partial q}, t) + \frac{\partial S}{\partial t} = 0. \quad (2)$$

Assume now that W is separable in the following manner: $W = p_\theta \theta + S_1(r) + S_2(z)$, and show that the quantity

$$\beta = b(z) + \frac{1}{2m}[S_2'(z)]^2 \quad (3)$$

has to be a constant. Finally, write down the solution for Hamilton's principal function on integral form.

(1b) What are the advantages of using the Hamilton-Jacobi formalism to solve a mechanical problem compared to other methods such as the Lagrangian method?