CLASSICAL MECHANICS TFY4345 - Exercise 12

(1a) Under a canonical transformation $(q_i, p_i) \rightarrow (Q_i, P_i)$ the Poisson bracket is invariant:

$$[u, v]_{q,p} = [u, v]_{Q,P} \tag{1}$$

Above, u and v are arbitrary functions. Show by a direct calculation that the above equation is satisfied for a harmonic oscillator, where $q = \sqrt{2P/m\omega}\sin Q$ and $p = m\omega q \cot Q$ with the definition $P = E/\omega$.

(1b) Explain why it can be useful to perform a canonical transformation, i.e. what kind of advantages does it offer in the analysis of a problem?

(2a) Find the relativistic transformation formulas for the magnetic field $\mathbf{B}$ and show that when $v \ll c$ one can write the transformation on vector form:

$$\mathbf{B}' = \mathbf{B} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}. \tag{2}$$

(2b) Using the above formula, explain the following: how does spin-orbit interaction for an electron occur when moving in the electric field of a nucleus even though there is no magnetic field generated by the nucleus?

(2c) A magnetohydrodynamic wave propagates in the x direction with a velocity $\mathbf{u}$. A strong external magnetic field $\mathbf{B}_0$ is present. The liquid is perfectly conducting, such that the electrical conductivity $\sigma \rightarrow \infty$. In the interior of the liquid, there exists an electric field $\mathbf{E} \simeq -\mathbf{u} \times \mathbf{B}_0$. Why? Assume non-relativistic motion.

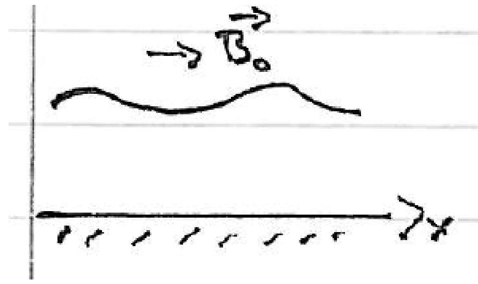

FIG. 1: (Color online). The system under consideration.