CLASSICAL MECHANICS TFY4345 - Exercise 11

(1a) The CO_2 molecule is given, with masses M and m. The deviations from equilibrium are written as $\eta_i = \sum_\alpha \Delta_{i\alpha}\Theta_\alpha$, where $\theta_\alpha = \mathrm{Re}\{C_\alpha \mathrm{e}^{-\mathrm{i}\omega_\alpha t}\}$ are the normal coordinates. Write V_{ij} and T_{ij} on matrix form.

(1b) Identify the eigenfrequencies ω_1, ω_s, and ω_a.

(1c) Find the cofactors $\Delta_{i\alpha}$, for $i = 1,2,3$, $\alpha = 1,s,a$. Use the normalization $\sum_{i,j=1}^{3} T_{ij}\Delta_{i\alpha}\Delta_{j\beta} = \delta_{\alpha\beta}$.

(1d) Describe in words what kind of motion of the molecule the different solutions correspond to.

(2) Find the velocity components (u_x, u_y, u_z) in the inertial system S expressed in terms of the corresponding components (u'_x, u'_y, u'_z) in S'. The system S' is moving with a speed v along the z-axis of S. Assume that a particle is instantaneously at rest in S', i.e. $\mathbf{u}' = 0$ at the time of the measurement, and that its acceleration in S' at the same time is $\mathbf{a}'$. Show that the acceleration components $a_i = du_i/dt$ in S are:

$$a_x = (1-\beta^2)a'_x \tag{1}$$

$$a_y = (1-\beta^2)a'_y \tag{2}$$

$$a_z = (1-\beta^2)^{3/2}a'_z. \tag{3}$$

(3) A radioactive ^{57}Co element is situated on the periphery of a rotating disk. The peripheral velocity is u. The radiation is received by an observer located in the center of the disk. Let ν^0 be the eigenfrequency of the radiation in the inertial system where the element is momentarily at rest. Find the frequency ν of the observed radiation.

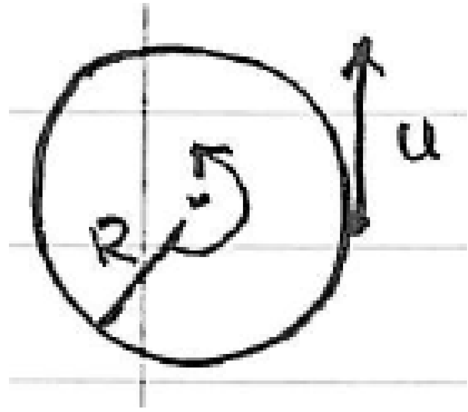

FIG. 1: (Color online). The system under consideration.