CLASSICAL MECHANICS TFY4345 - Exercise 10

A ridig strong rod with uniform mass density hangs in a stationary position from a fixed point at the equator, directed upwards, without being suspended at any point (the rod thus follows the rotation of Earth). The radius R of the Earth can be set to 6400 km.

(1a) What is the length L of the rod? Explain which physical principle you are using to solve the problem.

(1b) Imagine now that the rod still is fixed at the equator, but that its length L is shorter than the answer in a). What would happen with the rod? What happens if L is longer than the answer in a)?

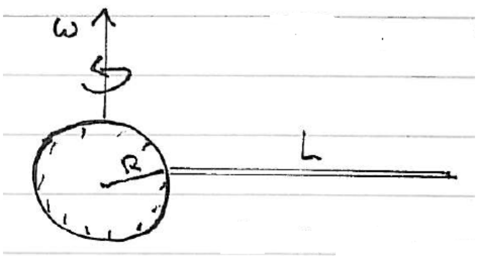

FIG. 1: (Color online). The system under consideration.

(2) A rigid body is symmetric around the x_3 axis. The moments of inertia are $I_1, I_2 = I_1$ and I_3. There are no external forces. Use the formulas

$$\omega_1 = \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\psi \tag{1}$$

$$\omega_2 = \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\psi \tag{2}$$

$$\omega_3 = \dot{\phi}\cos\theta + \dot{\psi} \tag{3}$$

to show that the rotational energy around the CM is:

$$T = \frac{1}{2}I_1(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta) + \frac{1}{2}I_3(\dot{\psi} + \dot{\phi}\cos\theta)^2 \tag{4}$$

Use Lagrangian theory to show that the canonical momentum components p_ϕ and p_ψ are constants, equal respectively to L_z (in the fixed lab-frame), and L_3. Find $\dot{\phi} = \dot{\phi}(\theta, L_z, L_3)$ and $\dot{\psi} = \dot{\psi}(\theta, L_z, L_3)$ and also the energy E expressed by means of $\theta, \dot{\theta}, L_z$, and L_3.