CLASSICAL MECHANICS TFY4345 - Exercise 1

(1a) Use energy conservation to calculate the escape velocity of a massive object from the Earth. *Hint:* choose the reference level for potential energy such that $V = 0$ as $r \to \infty$.

(1b) Show that for a particle with mass m, the equation of motion $\mathbf{F} = m\dot{\mathbf{v}}$ implies that the kinetic energy T must satisfy $dT/dt = \mathbf{F} \cdot \mathbf{v}$. Also show that if the particle has variable mass, the corresponding expression becomes $d(mT)/dt = \mathbf{F} \cdot \mathbf{p}$.

(1c) Assume a two-particle system in one dimension, with potential $V(x_1, x_2) = (k/2)(x_1 - x_2 - l)^2$. Here, k and l are constants while x_1 and x_2 are the particle coordinates. Introduce the relative coordinate $x = x_1 - x_2$ and the centre of mass coordinate $R = (m_1x_1 + m_2x_2)/(m_1 + m_2)$. Show that the equations of motion are:

$$\ddot{R} = 0,\ \ddot{x} = -(k/\mu)(x - l) \tag{1}$$

where $\mu = m_1m_2/(m_1 + m_2)$ is the reduced mass. Find $(x - l)$ as a function of t when $x(t = t_0) = l$. Express the solution in terms of the energy E and t. What happens in the limit $k \to \infty$?